Dyson-Schwinger Equations
and Quantum Chromodynamics

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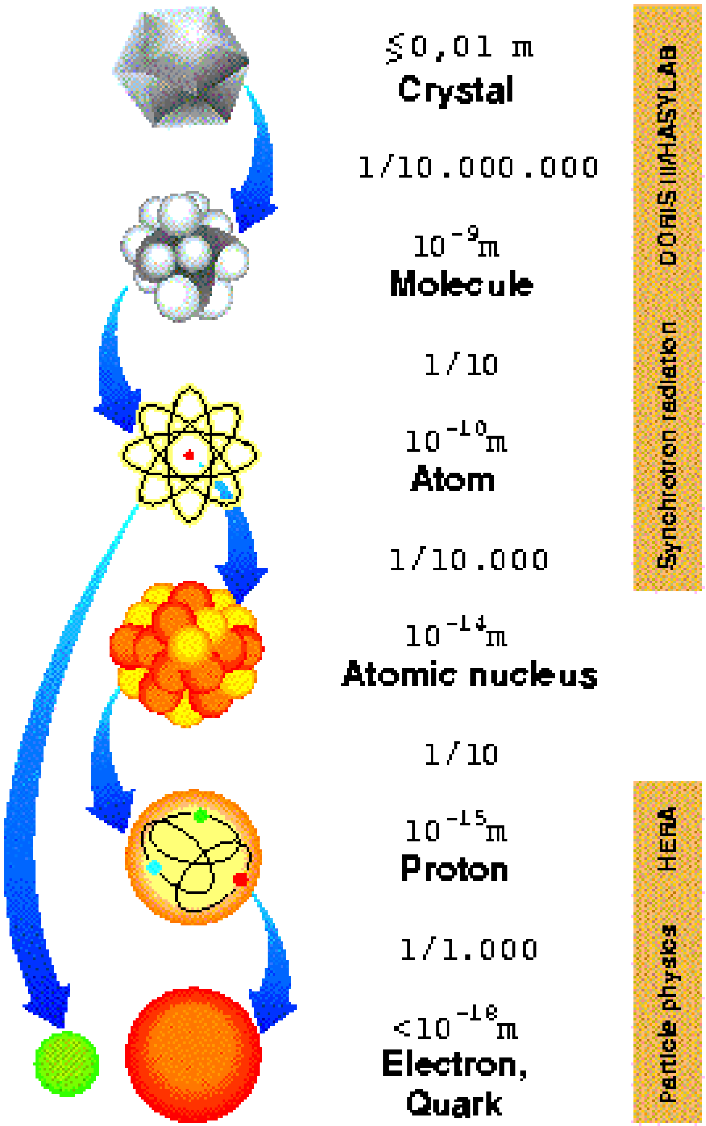
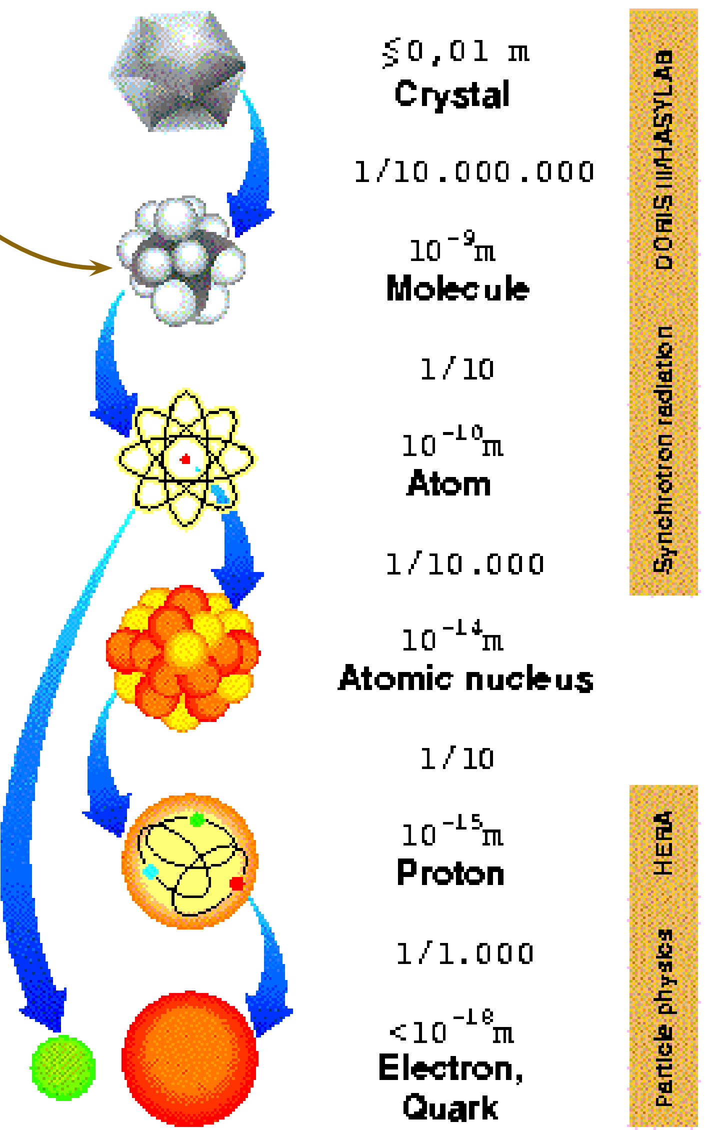
http://www.phy.anl.gov/theory/staff/cdr.html

Hadron Physics



Hadron Physics

Molecular Physics
Scale = nm

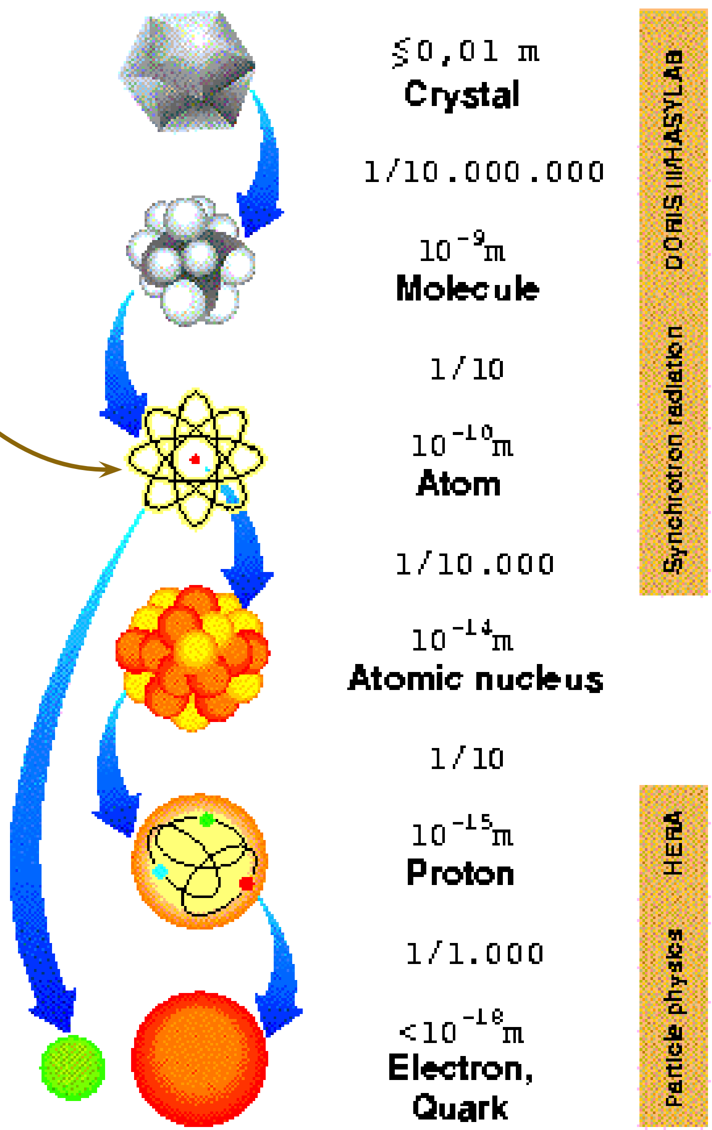


First Contents Back Conclusion

Hadron Physics

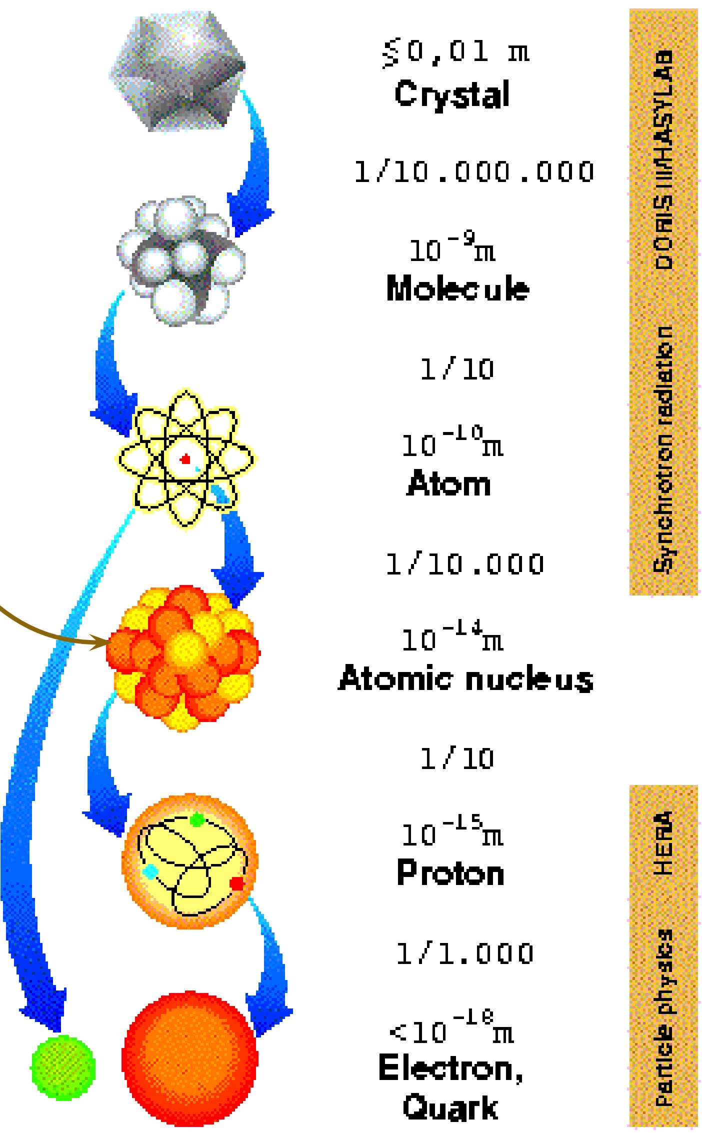
Atomic Physics

Scale = Å

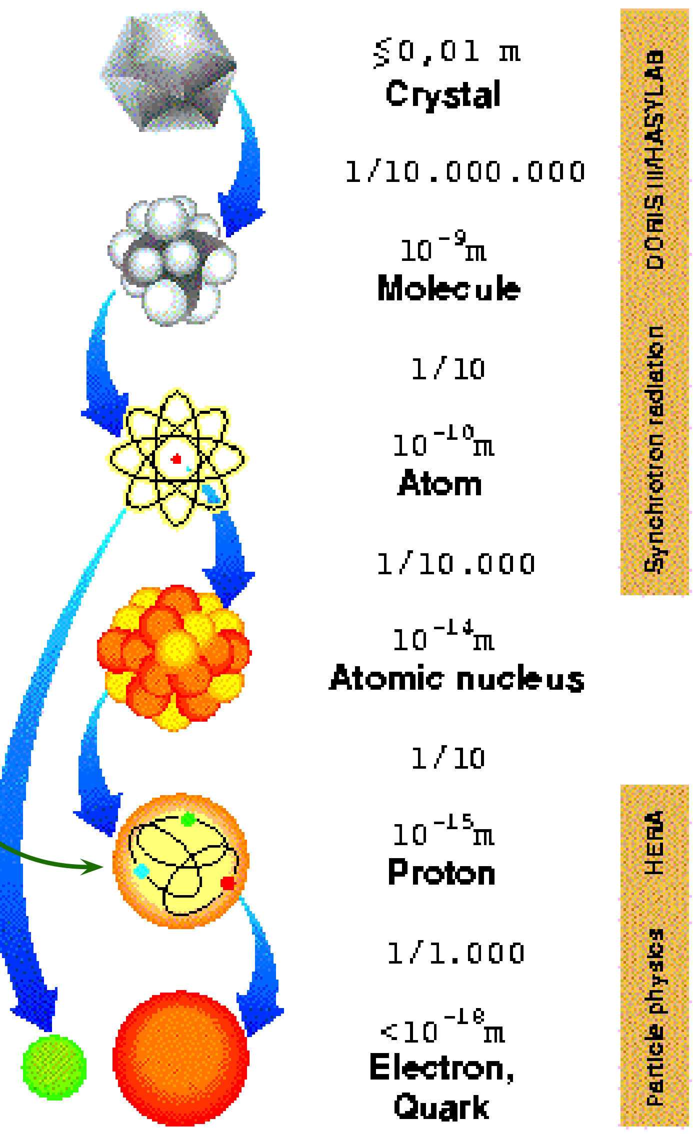


Hadron Physics

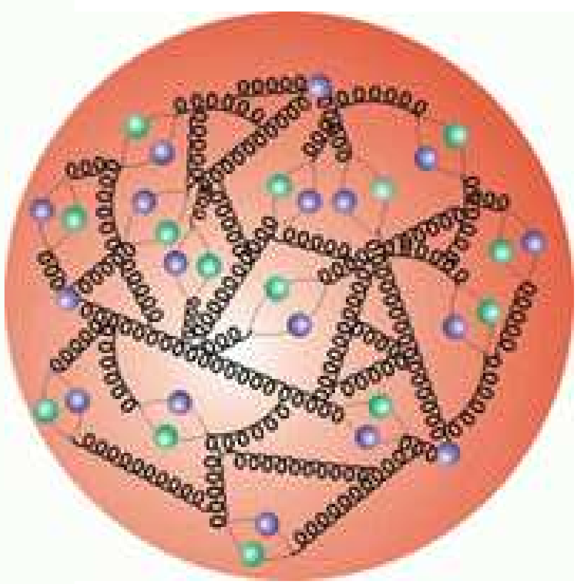
Nuclear Physics
Scale = 10 fm

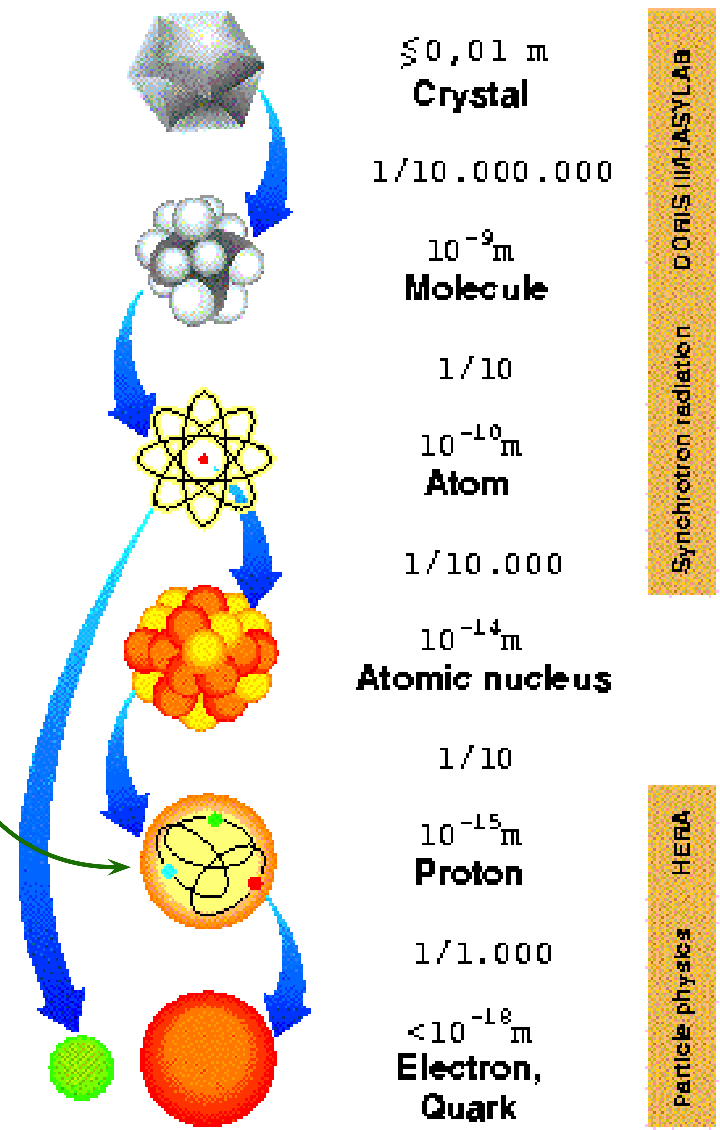


Hadron Physics
Scale = 1 fm

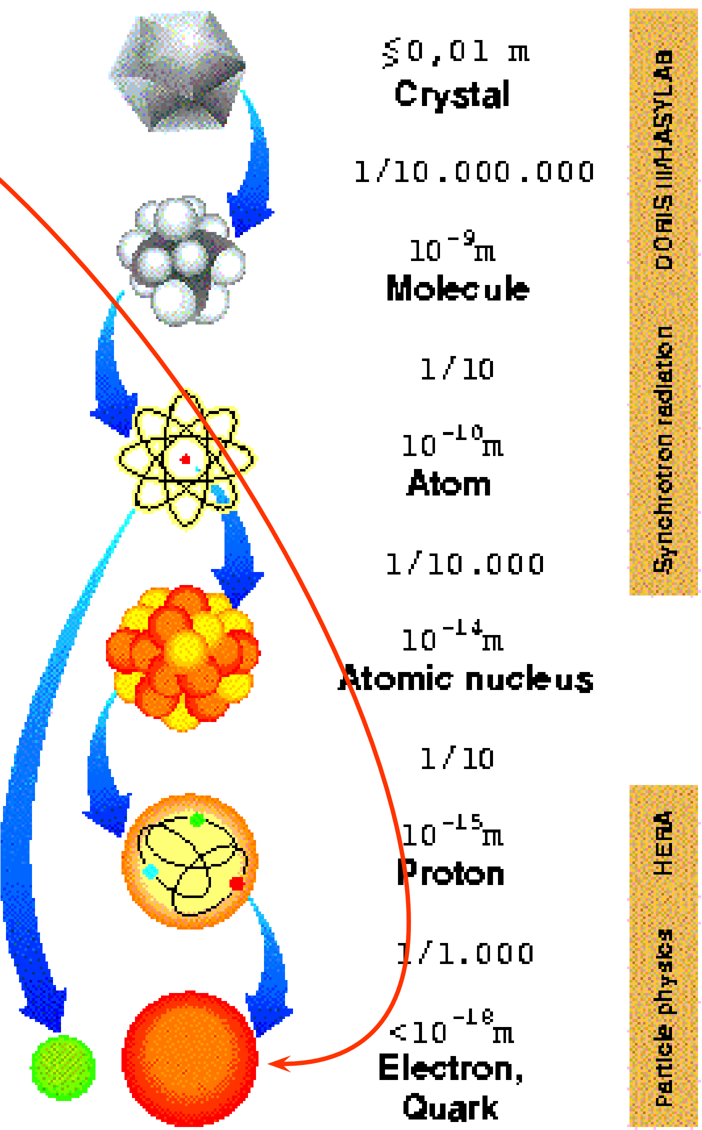


# Hadron Physics
Scale = 1 fm





Meta-Physics
Scale = Limited only
by Theorists
Imagination



Nucleon . . . 2 Key Hadrons
$=$ Proton and Neutron

First Contents Back Conclusion

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First Contents Back Conclusion

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  - Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
    Quark discovery via $e^- p$-scattering at SLAC in 1968
    – the elementary quanta of Quantum Chromo-dynamics

- Action, in terms of local Lagrangian density:

$$S[A^a_\mu, \bar{q}, q] = \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + \frac{1}{2\xi} \partial_\mu A^a_\mu(x)\, \partial_\nu A^a_\nu(x) + \bar{q}(x) \left[\gamma_\mu D_\mu + M\right] q(x) \right\} \tag{1}$$

  - Chromomagnetic Field Strength Tensor – $\partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g f^{abc} A^b_\mu(x) A^c_\nu(x)$
  - Covariant Derivative – $D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} A^a_\mu(x)$
  - Current-quark Mass matrix: $\begin{pmatrix} m_u & 0 & 0 & \ldots \\ 0 & m_d & 0 & \ldots \\ 0 & 0 & m_s & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}$

- Understanding JLab Observables means knowing all that this Action predicts.
- Perturbation Theory (asymptotic freedom) is not enough!
  - Bound states are not perturbative
  - Confinement is not perturbative
  - DCSB is not perturbative

Euclidean Metric

- Almost all nonperturbative studies in relativistic quantum field theory employ a Euclidean Metric. (NB. Remember the Wick Rotation?)
- It is possible to view the Euclidean formulation of a quantum field theory as definitive; e.g.,
  - Symanzik, K. (1963) in *Local Quantum Theory* (Academic, New York) edited by R. Jost.
  - Streater, R.F. and Wightman, A.S. (1980), *PCT, Spin and Statistics, and All That* (Addison-Wesley, Reading, Mass, 3rd edition).
  - Glimm, J. and Jaffee, A. (1981), *Quantum Physics. A Functional Point of View* (Springer-Verlag, New York).
  - Seiler, E. (1982), *Gauge Theories as a Problem of Constructive Quantum Theory and Statistical Mechanics* (Springer-Verlag, New York).
- That decision is crucial when a consideration of nonperturbative effects becomes important. In addition, the discrete lattice formulation in Euclidean space has allowed some progress to be made in attempting to answer existence questions for interacting gauge field theories.
  - A lattice formulation is impossible in Minkowski space – the integrand is not non-negative and hence does not provide a probability measure.

Euclidean Metric: Transcription Formulae

- To make clear our conventions: for 4-vectors a, b: $a \cdot b := a_\mu \, b_\nu \, \delta_{\mu\nu} := \sum_{i=1}^{4} a_i \, b_i$,

  Hence, a spacelike vector, Q_μ, has $Q^2 > 0$.
- Dirac matrices:
  - Hermitian and defined by the algebra $\{\gamma_\mu, \gamma_\nu\} = 2\, \delta_{\mu\nu}$;
  - we use $\gamma_5 := -\gamma_1\gamma_2\gamma_3\gamma_4$, so that $\mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = -4\, \varepsilon_{\mu\nu\rho\sigma}$, $\varepsilon_{1234} = 1$.
  - The Dirac-like representation of these matrices is:

$$\vec{\gamma} = \left(\begin{array}{cc} 0 & -i\vec{\tau} \\ i\vec{\tau} & 0 \end{array} \right), \; \gamma_4 = \left(\begin{array}{cc} \tau^0 & 0 \\ 0 & -\tau^0 \end{array} \right), \tag{2}$$

where the 2×2 Pauli matrices are:

$$\tau^0 = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right), \; \tau^1 = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \; \tau^2 = \left(\begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right), \; \tau^3 = \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right). \tag{3}$$

Euclidean Metric: Transcription Formulae

- It is possible to derive every equation introduced above assuming certain analytic properties of the integrands. However, the derivations can be sidestepped using the following *transcription rules*:

Configuration Space	Momentum Space
1. $\int^M d^4x^M \to -i\int^E d^4x^E$	1. $\int^M d^4k^M \to i\int^E d^4k^E$
2. $\not\partial \to i\gamma^E\cdot\partial^E$	2. $\not k \to -i\gamma^E\cdot k^E$
3. $\not A \to -i\gamma^E\cdot A^E$	3. $\not A \to -i\gamma^E\cdot A^E$
4. $A_\mu B^\mu \to -A^E\cdot B^E$	4. $k_\mu q^\mu \to -k^E\cdot q^E$
5. $x^\mu\partial_\mu \to x^E\cdot\partial^E$	5. $k_\mu x^\mu \to -k^E\cdot x^E$

- These rules are valid in perturbation theory; i.e., the correct Minkowski space integral for a given diagram will be obtained by applying these rules to the Euclidean integral: they take account of the change of variables and rotation of the contour. However, for diagrams that represent DSEs which involve dressed n-point functions, whose analytic structure is not known *a priori*, the Minkowski space equation obtained using this prescription will have the right appearance but it's solutions may bear no relation to the analytic continuation of the solution of the Euclidean equation. Any such differences will be nonperturbative in origin.

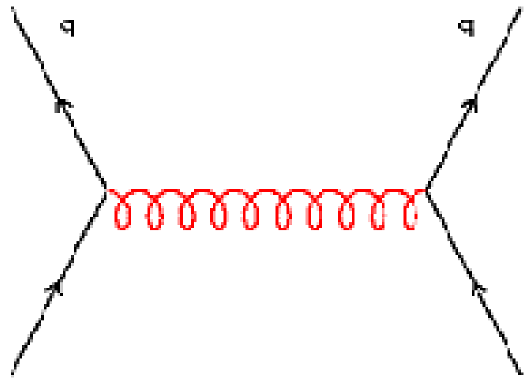
What is QCD?

What is QCD?

- Gauge Theory:

  Interactions Mediated by massless vector bosons

Feynman Diagram of Quark—Quark Scattering



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Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

Argonne NATIONAL LABORATORY

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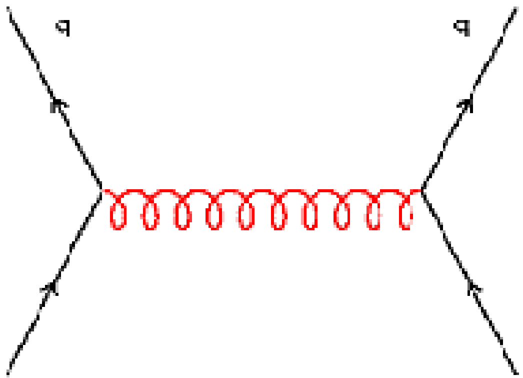

- Similar interaction in QED

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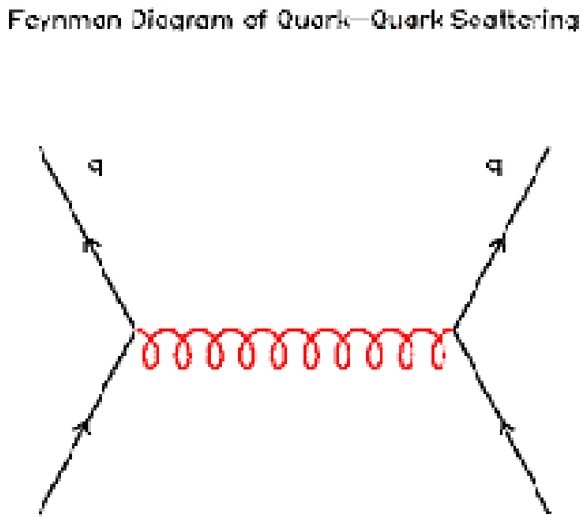
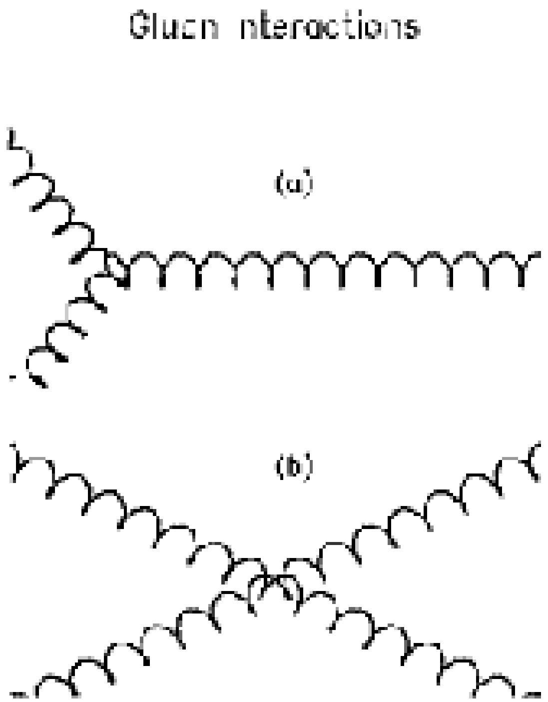




- Similar interaction in QED

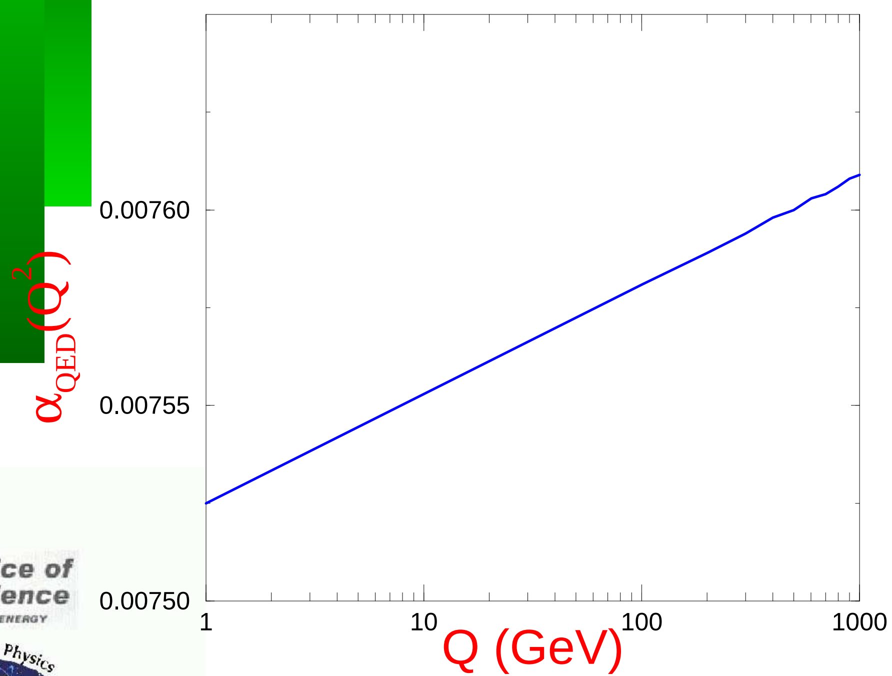
- Special Feature of QCD – gluon self-interactions

  Completely Change the Character of the Theory

QED cf. QCD

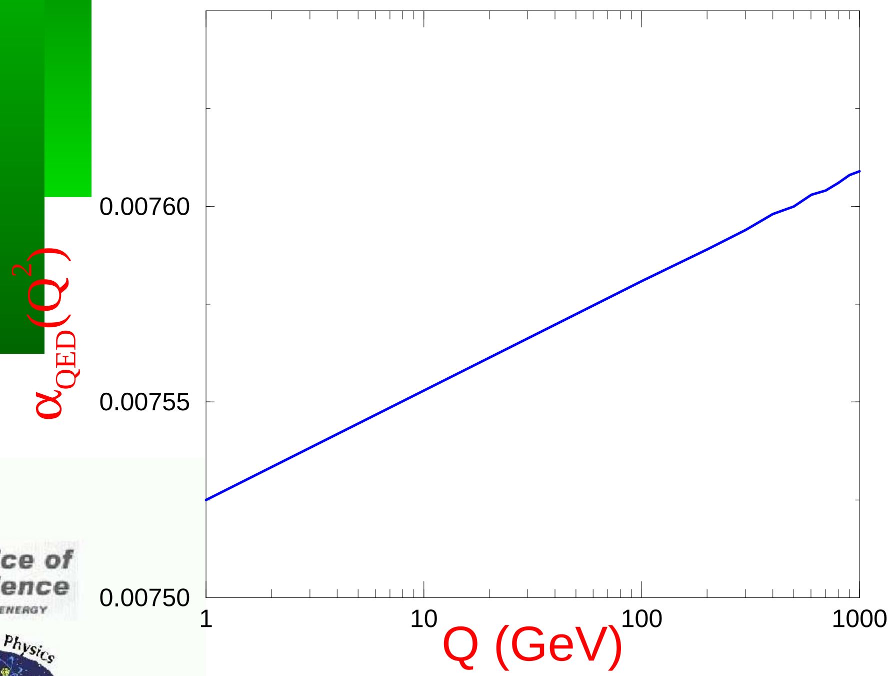
First Contents Back Conclusion

QED cf. QCD

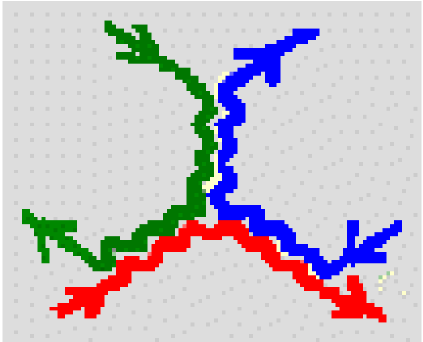


$$\alpha_{\mathrm{QED}} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

QED cf. QCD



Add three-gluon interaction



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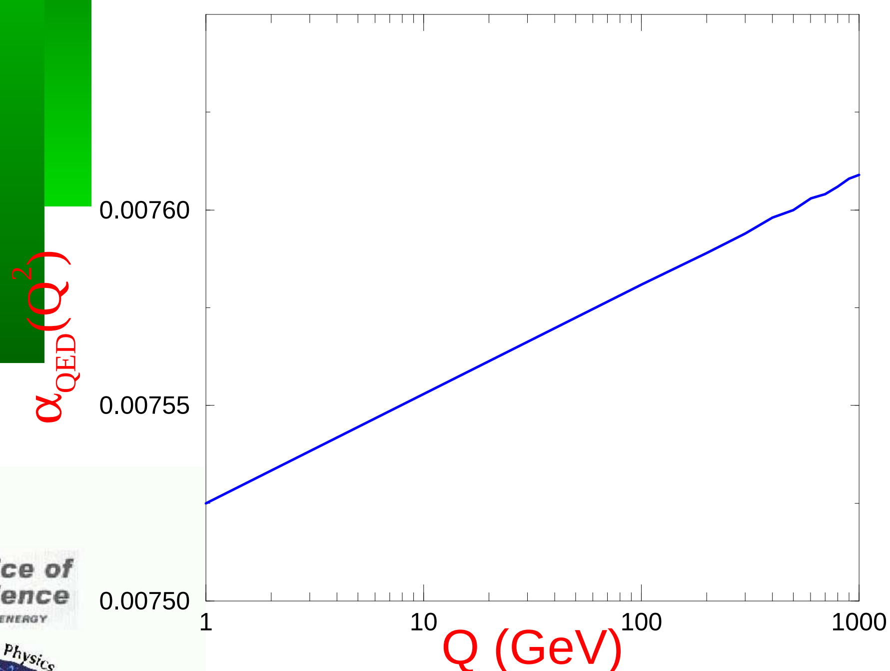


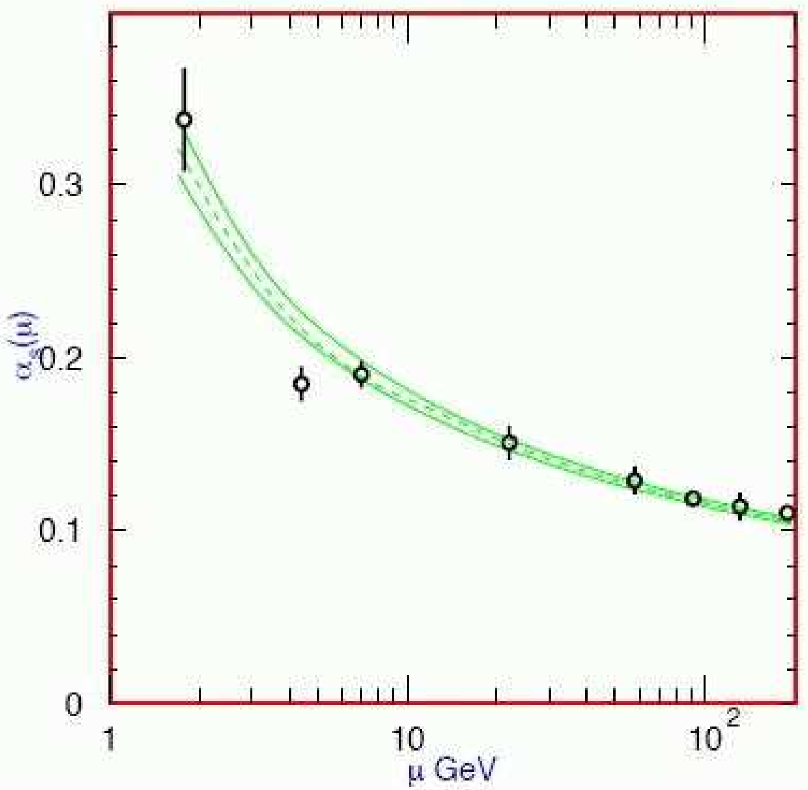


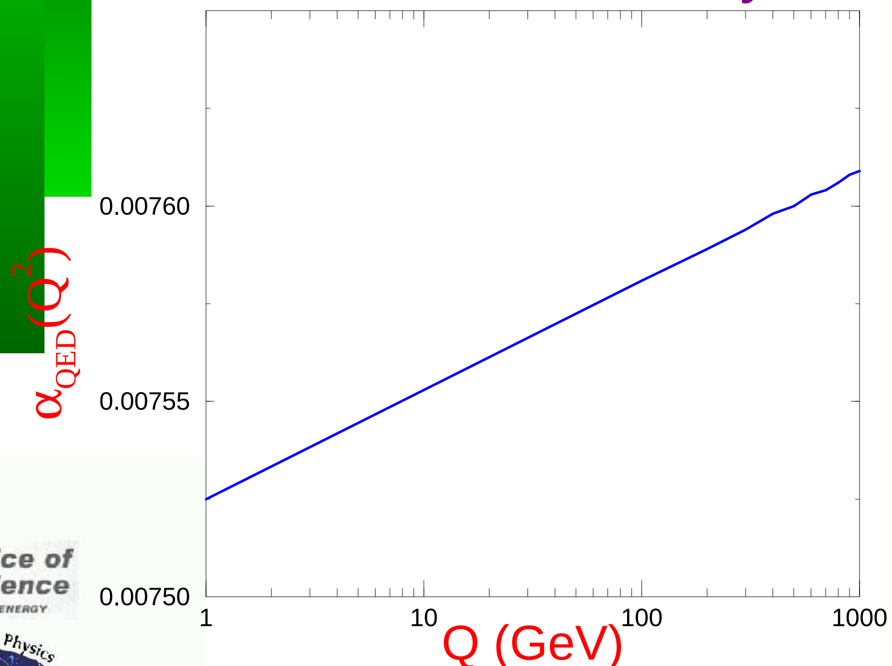
Figure 9.2: Summary of the values of $\alpha_s(\mu)$ at the values of μ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing μ. The data are, in increasing order of μ, τ width, Υ decays, deep inelastic scattering, e^+e^- event shapes at 22 GeV from the JADE data, shapes at TRISTAN at 58 GeV, Z width, and e^+e^- event shapes at 135 and 189 GeV.

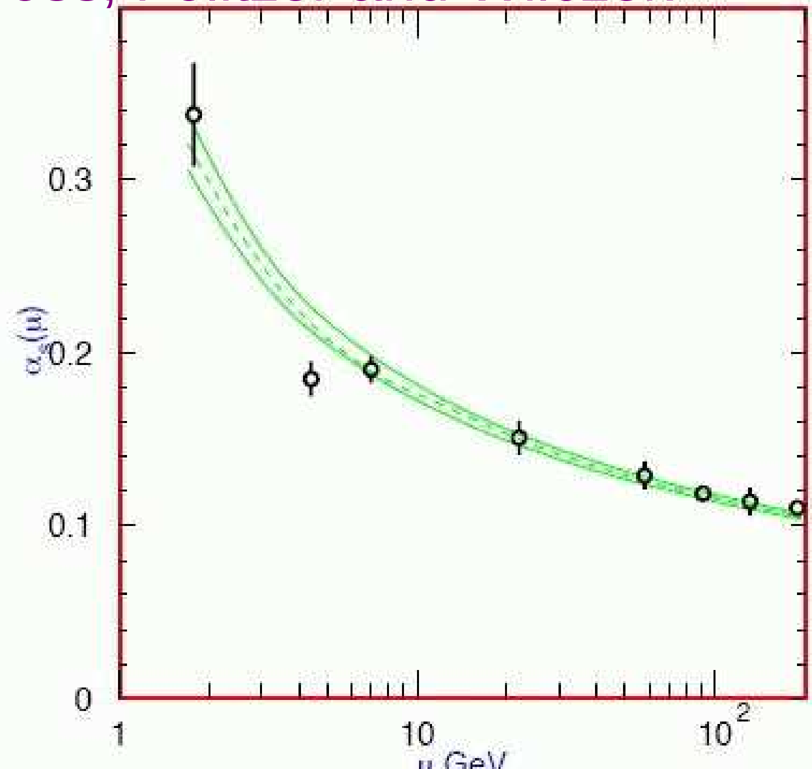
$$\alpha_{\rm QED} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

$$\alpha_{\rm QCD} = \frac{12\pi}{(33 - 2N_f)\ln\left(Q^2/\Lambda^2\right)}$$

QED cf. QCD

2004 Nobel Prize in Physics: Gross, Politzer and Wilczek





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First Contents Back Conclusion

Quarks and Nuclear Physics

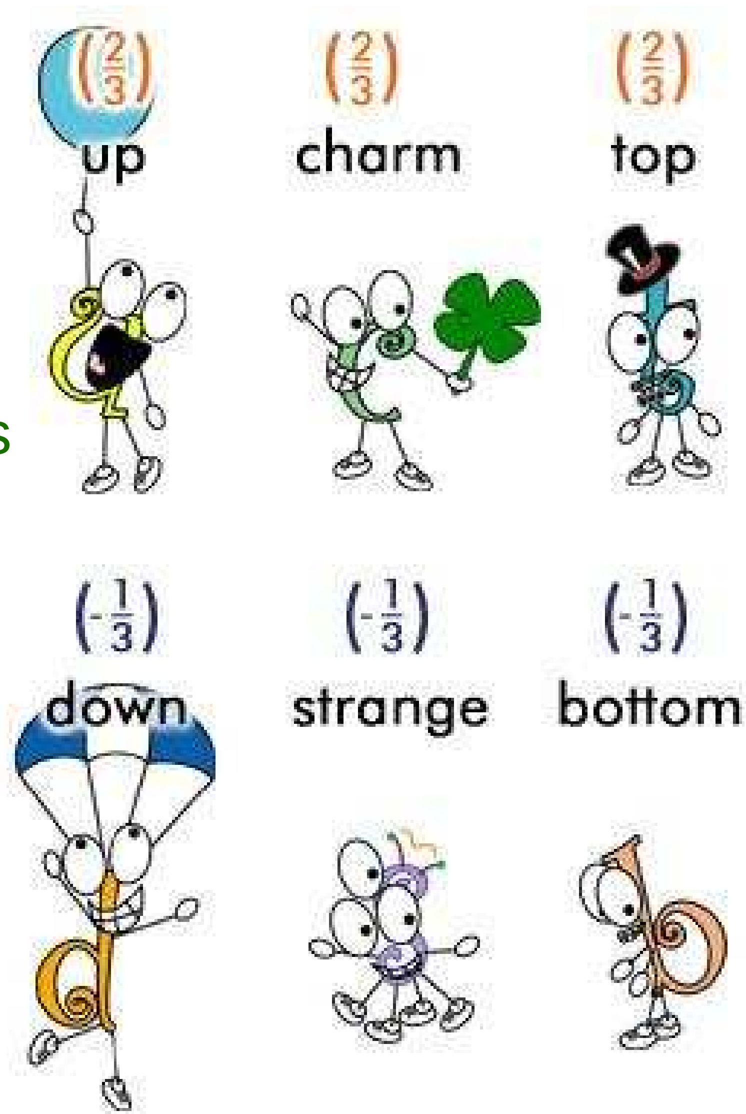
First Contents Back Conclusion

Quarks and Nuclear Physics

Standard Model

of Particle Physics

Six Flavours

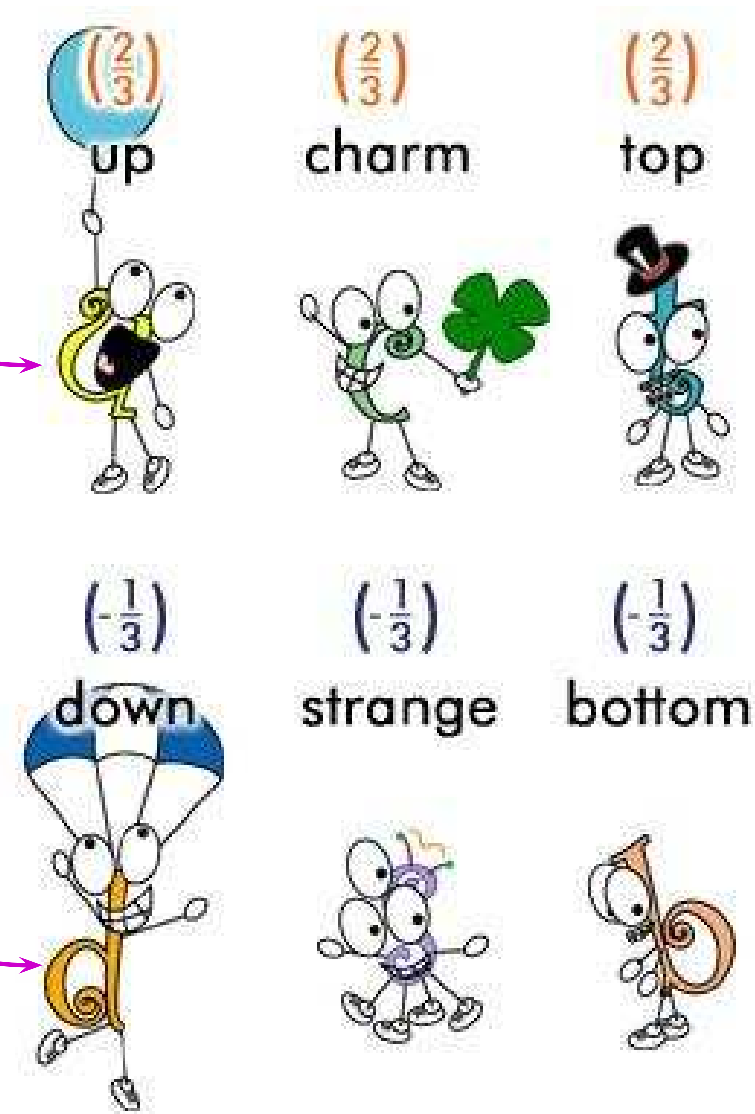


Real World

Normal Matter . . .

Only Two Light

Flavours Active



Quarks and Nuclear Physics

Real World

Normal Matter . . .

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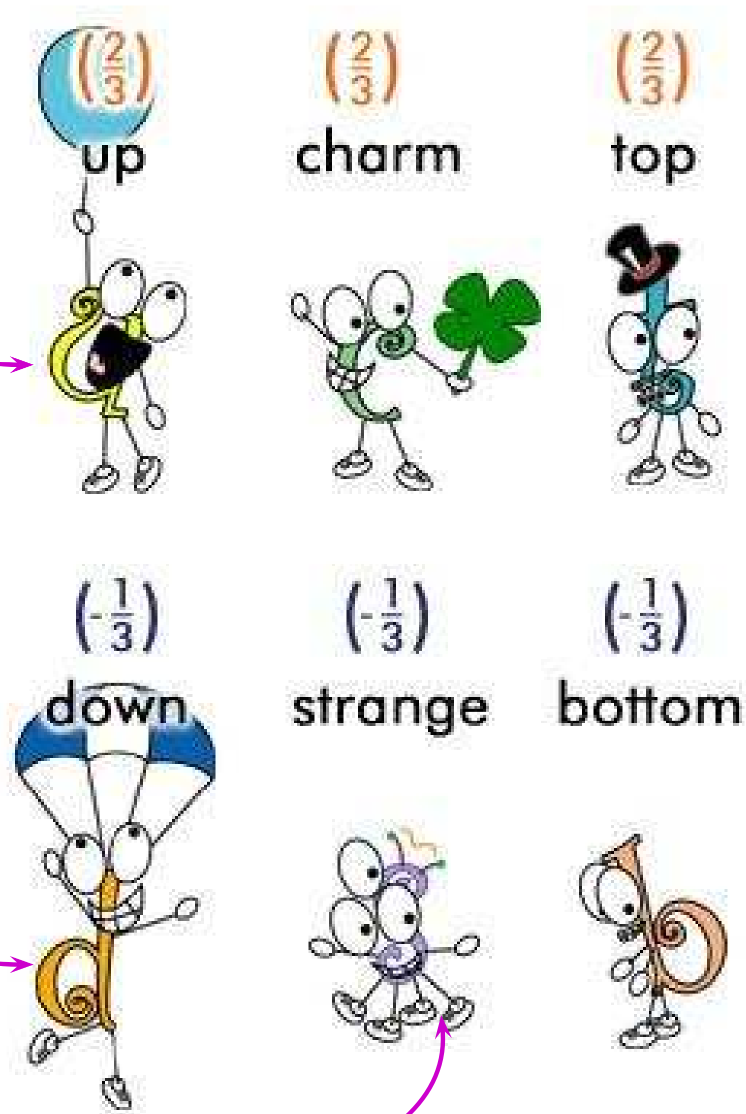
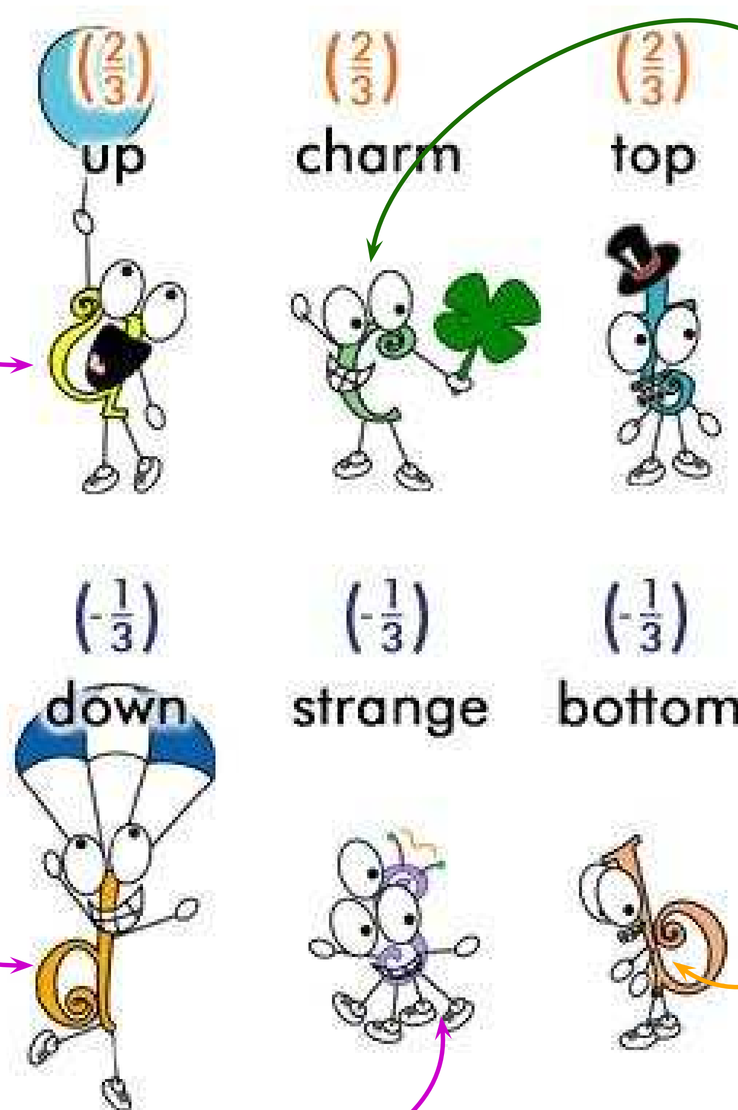
or, perhaps, three



Quarks and Nuclear Physics



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For numerous good reasons, much research also focuses on accessible heavy-quarks

Quarks and Nuclear Physics

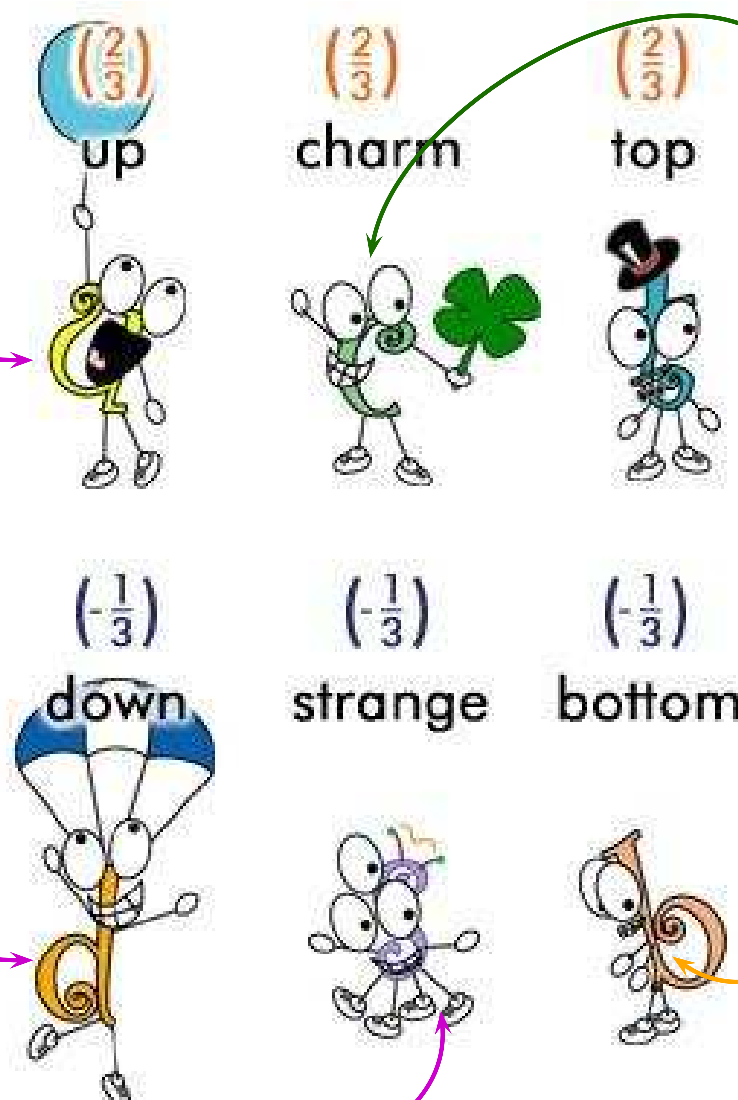
Nevertheless, I will focus primarily on the light-quarks.

Real World

Normal Matter . . .

Only Two Light Flavours Active

or, perhaps, three



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Simple Picture

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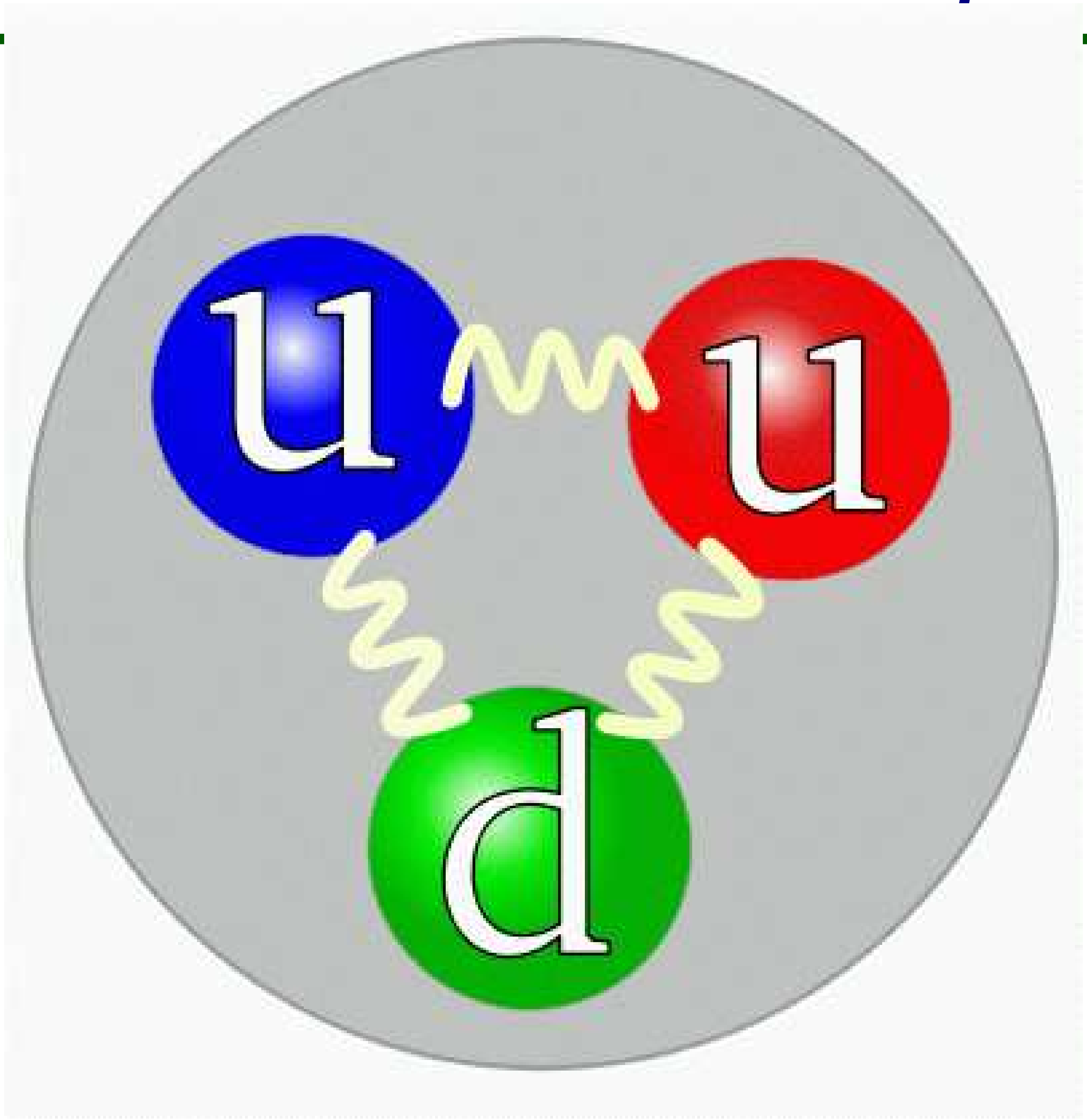
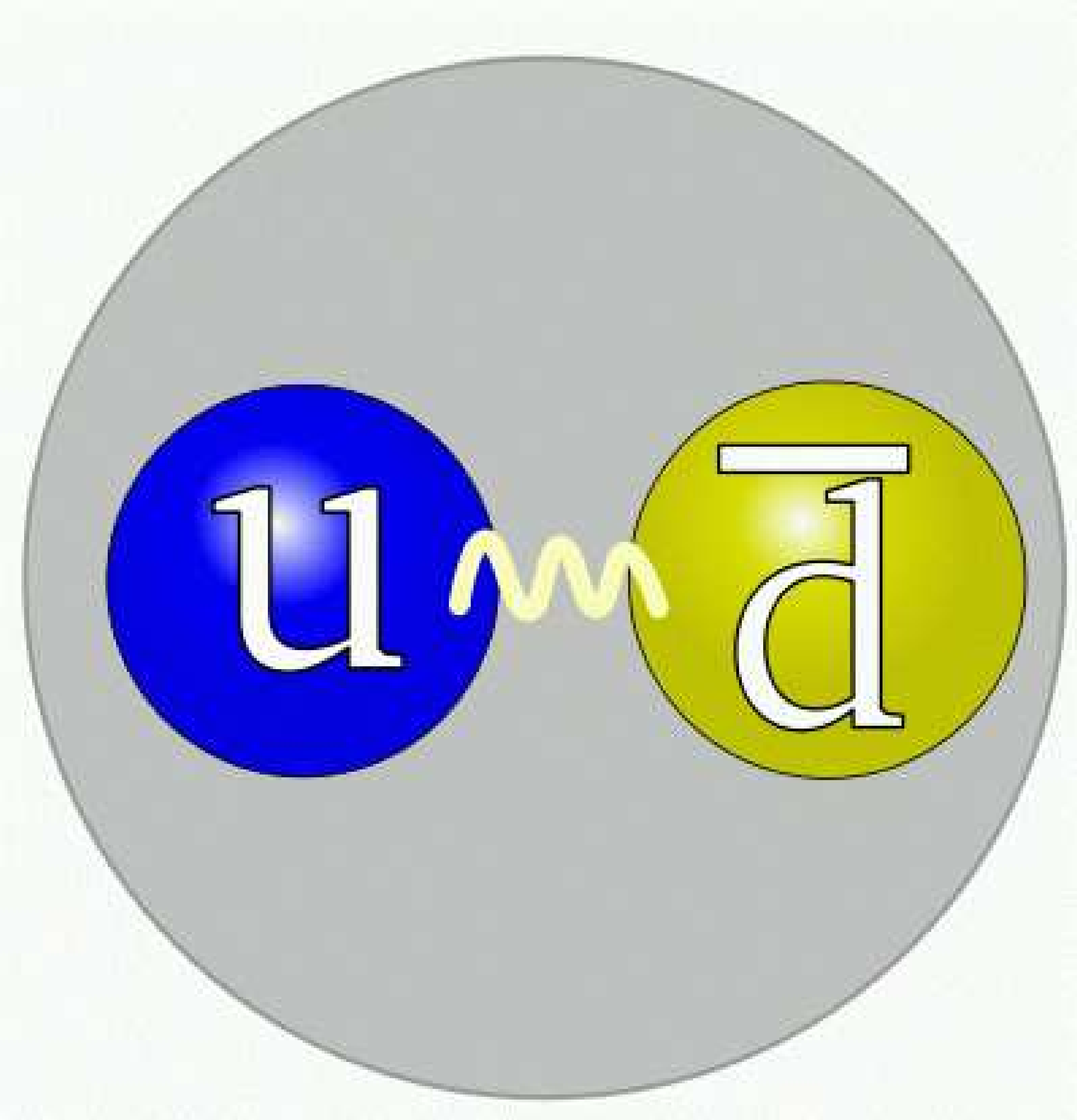


PROTON

Simple Picture



PION

Study Structure via Nucleon Form Factors

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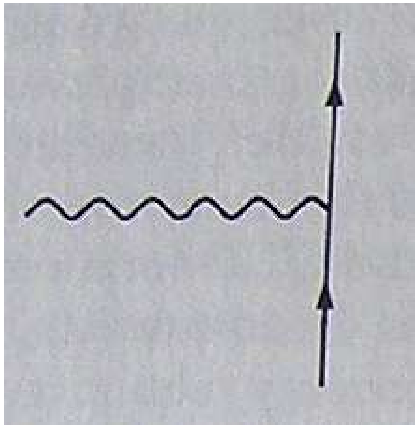
- Electron's relativistic electromagnetic current:

$$\begin{aligned} j_\mu(P',P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\ Q = P' - P \\ &= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P) \end{aligned}$$



First Contents Back Conclusion

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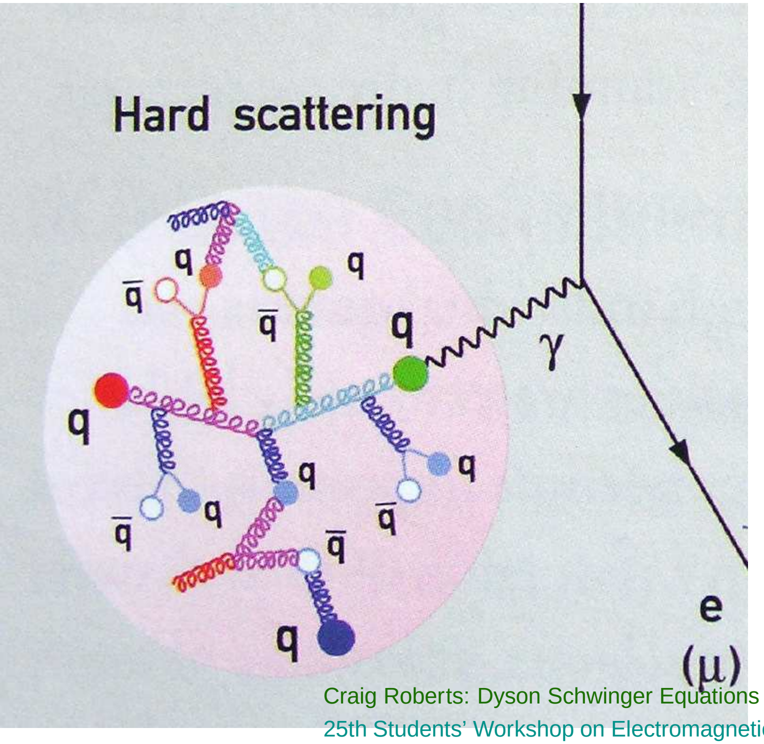

First Contents Back Conclusion

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$$G_E(Q^2)=F_1(Q^2)-\frac{Q^2}{4M^2}F_2(Q^2)\,,\ G_M(Q^2)=F_1(Q^2)+F_2(Q^2)\,.$$

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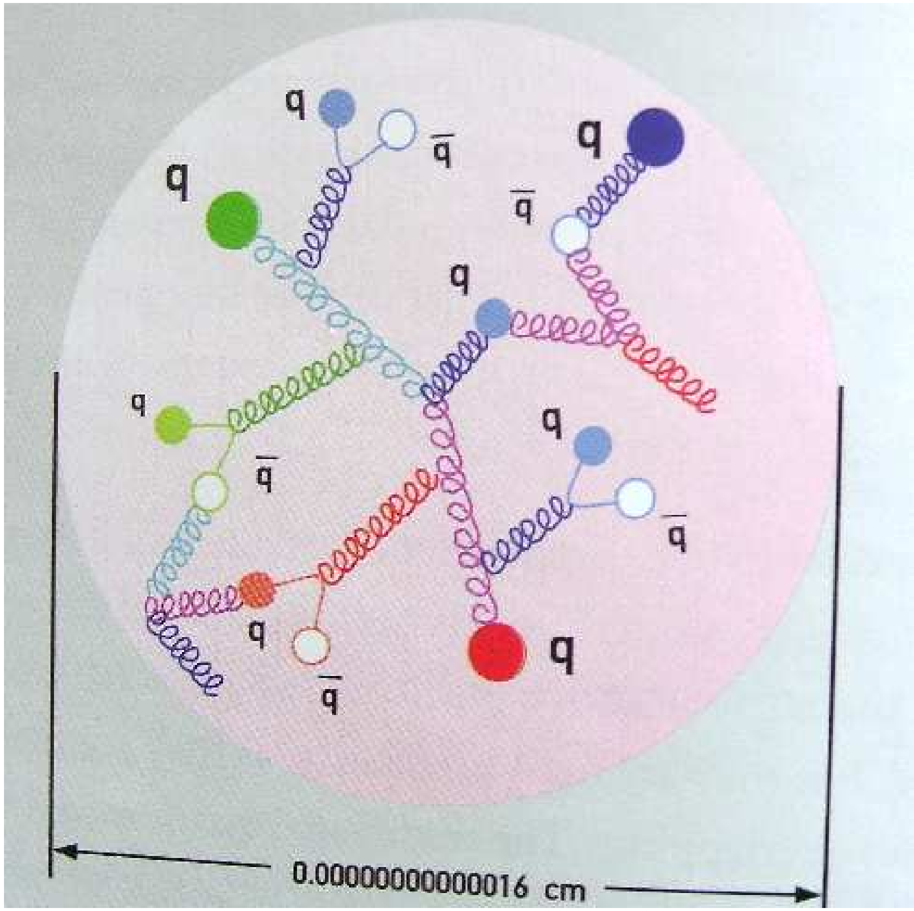
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\; G_M(Q^2) = F_1(Q^2) + F_2(Q^2)\,.$$

Point-particle: $\boldsymbol{F_2 \equiv 0 \Rightarrow G_E \equiv G_M}$

NSAC Long Range Plan

A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD



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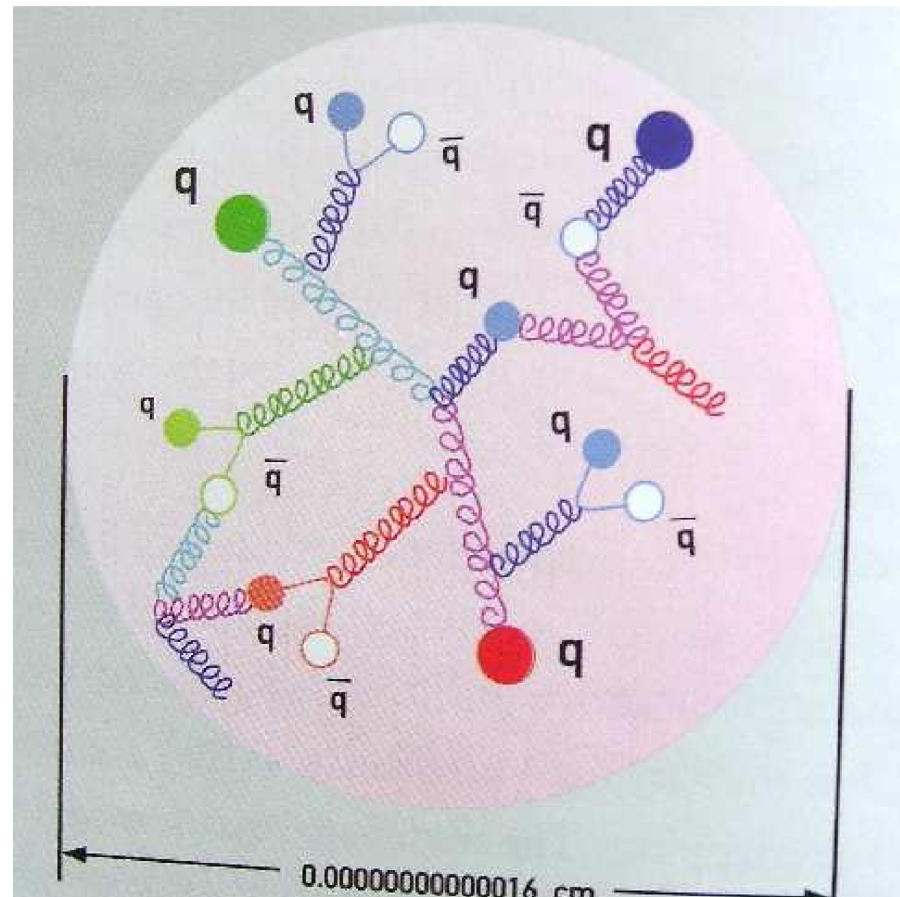

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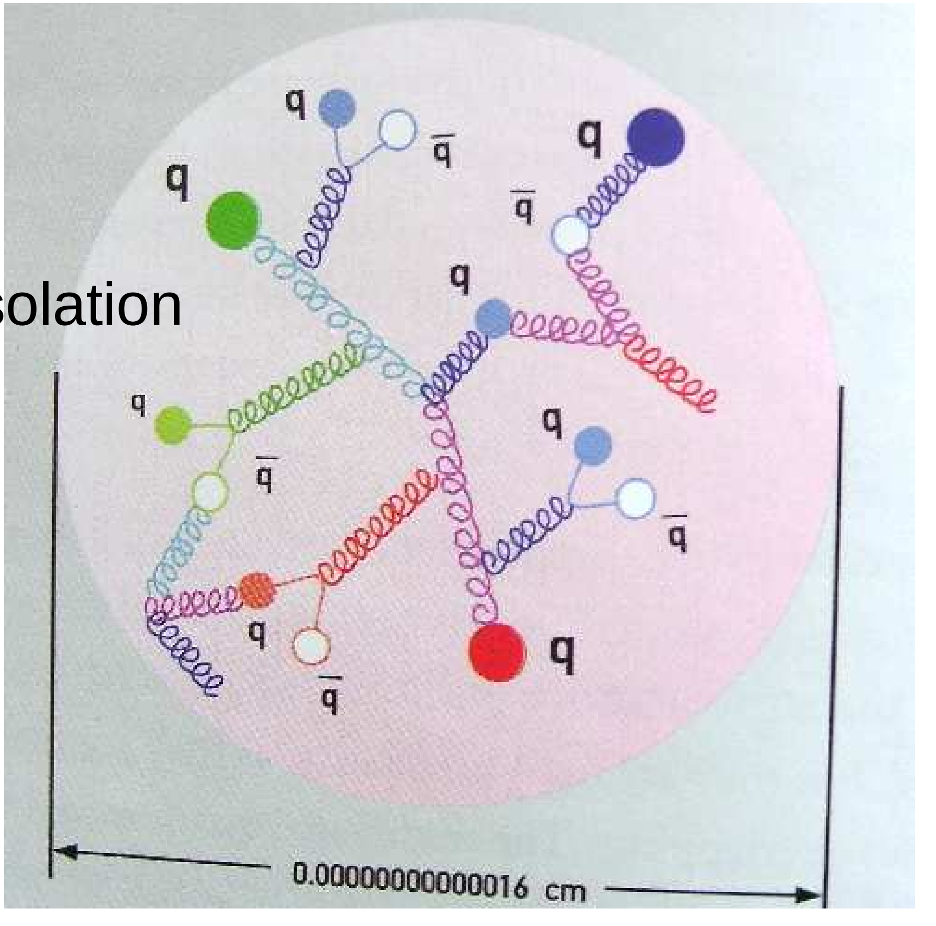

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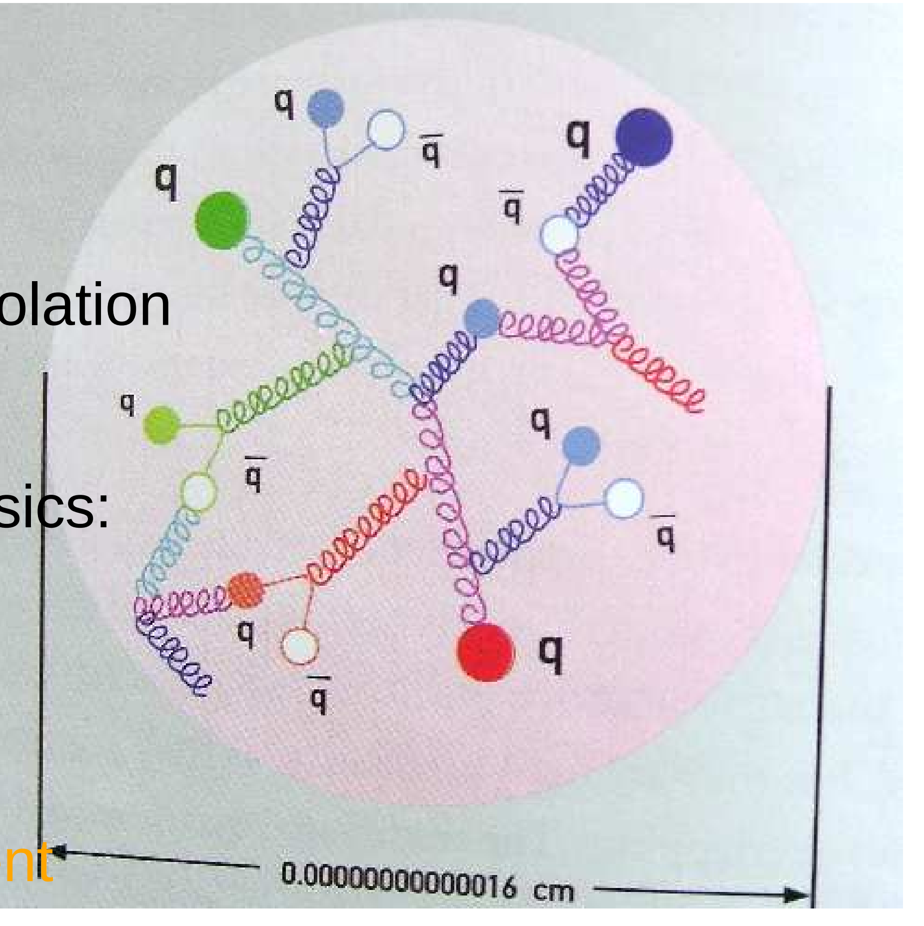
So, what's the problem?

- Confinement
  – No quark ever seen in isolation
- Weightlessness
  – 2004 Nobel Prize in Physics:
  Mass of $u-$ & $d-$quarks,
  each just 5 MeV;
  Proton Mass is 940 MeV
  $\Rightarrow$ No Explanation Apparent
  for 98.4 % of Mass



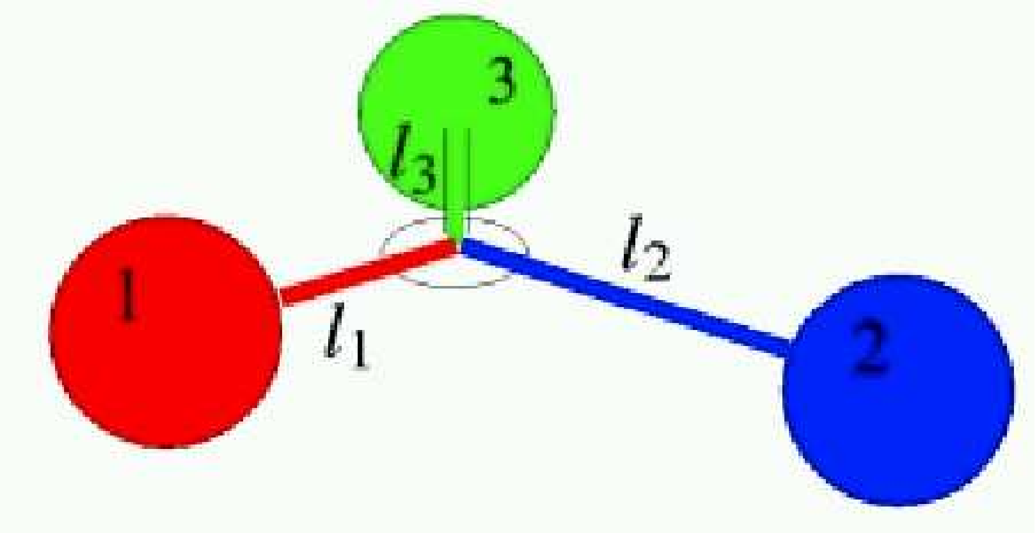
Meson Spectrum

LIGHT UNFLAVORED ($S = C = B = 0$)	$I^G(J^{PC})$		$I^G(J^{PC})$	STRANGE ($S = \pm 1$, $C = B = 0$)	$I(J^P)$
• $\pi^{\pm}$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$	• $K^{\pm}$	$1/2(0^-)$
• π^0 140 MeV	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$	• K^0	$1/2(0^-)$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$	• K^0_S	$1/2(0^-)$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$	• K^0_L	$1/2(0^-)$
• $\rho(770)$ 770	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$	$K^*_0(800)$	$1/2(0^+)$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$	• $K^*(892)$	$1/2(1^-)$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$	• $K_1(1270)$	$1/2(1^+)$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$	• $K_1(1400)$	$1/2(1^+)$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$	• $K^*(1410)$	$1/2(1^-)$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$	• $K^*_0(1430)$	$1/2(0^+)$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$	• $K^*_2(1430)$	$1/2(2^+)$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$	$K(1460)$	$1/2(0^-)$
• $a_1(1260)$	$1^-(1^{++})$	$\rho(1900)$	$1^+(1^{--})$	$K_2(1580)$	$1/2(2^-)$
• $f_2(1270)$	$0^+(2^{++})$	$f_2(1910)$	$0^+(2^{++})$	$K(1630)$	$1/2(?^?)$
• $f_1(1285)$	$0^+(1^{++})$	• $f_2(1950)$	$0^+(2^{++})$	$K_1(1650)$	$1/2(1^+)$
• $\eta(1295)$	$0^+(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$	• $K^*(1680)$	$1/2(1^-)$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$	• $K_2(1770)$	$1/2(2^-)$

First Contents Back Conclusion

Modern Miracles in Hadron Physics

- proton = three constituent quarks



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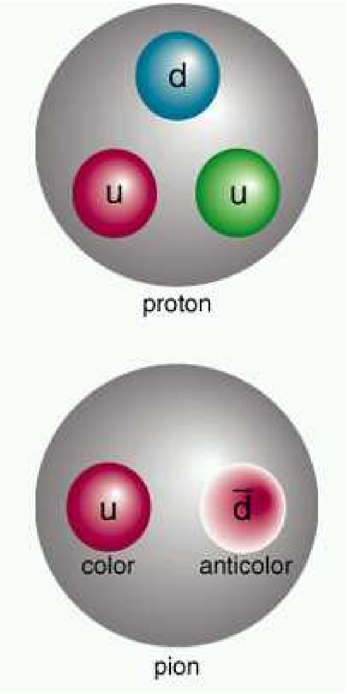

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- Another meson:
  . . . . $M_\rho = 770\,$MeV . . . . No Surprises Here

First | Contents | Back | Conclusion

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Dichotomy of Pion
– *Goldstone Mode* and *Bound state*

First Contents Back Conclusion

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Highly Nontrivial

First Contents Back Conclusion

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First Contents Back Conclusion

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First Contents Back Conclusion

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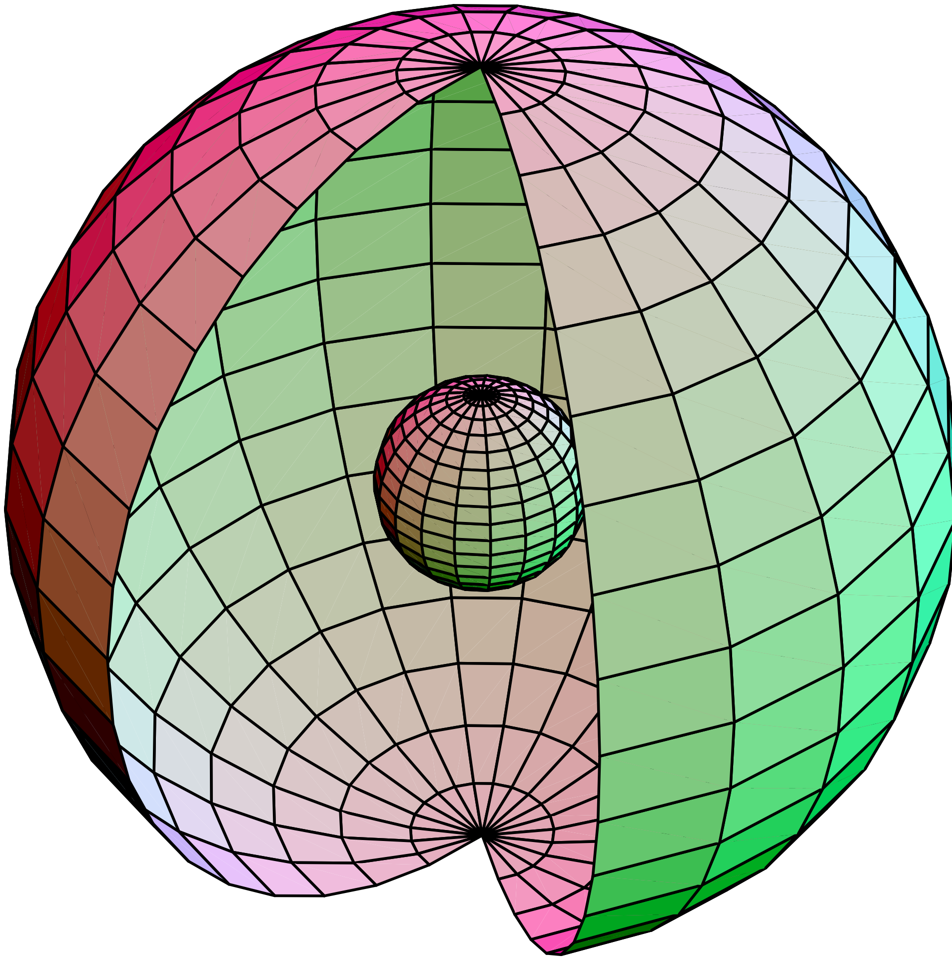
  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

Intranucleon Interaction

Intranucleon Interaction



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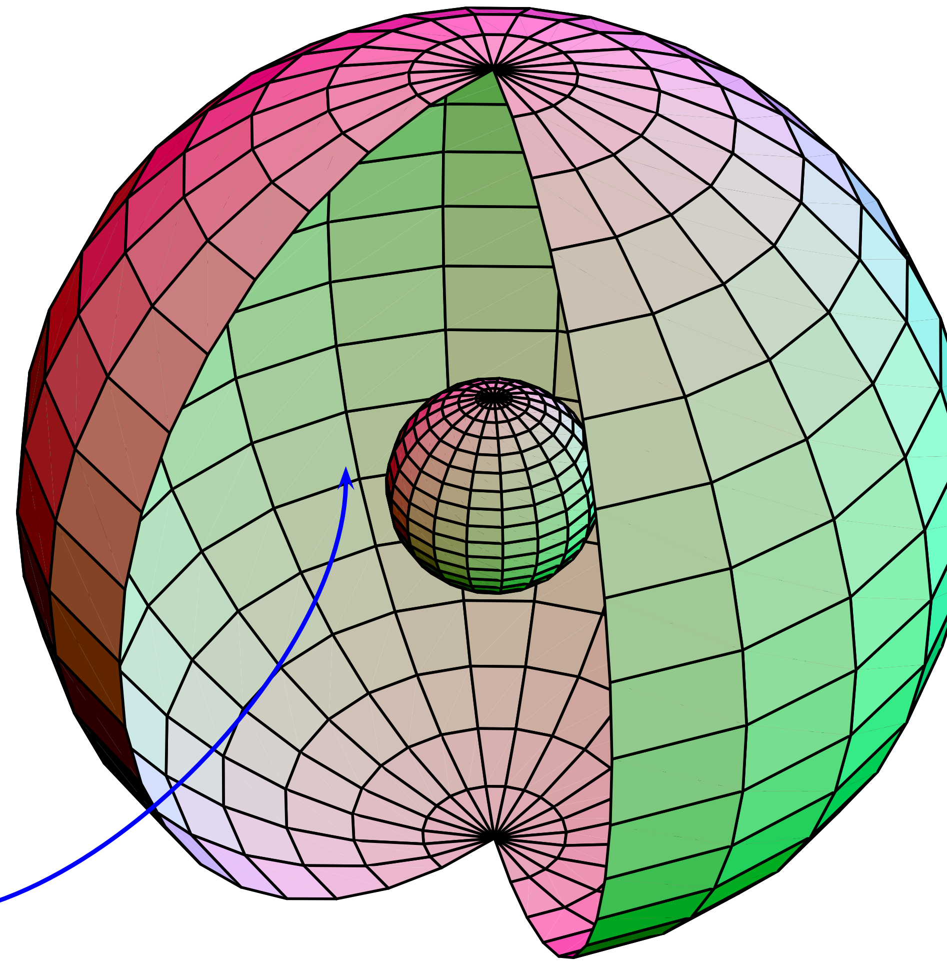
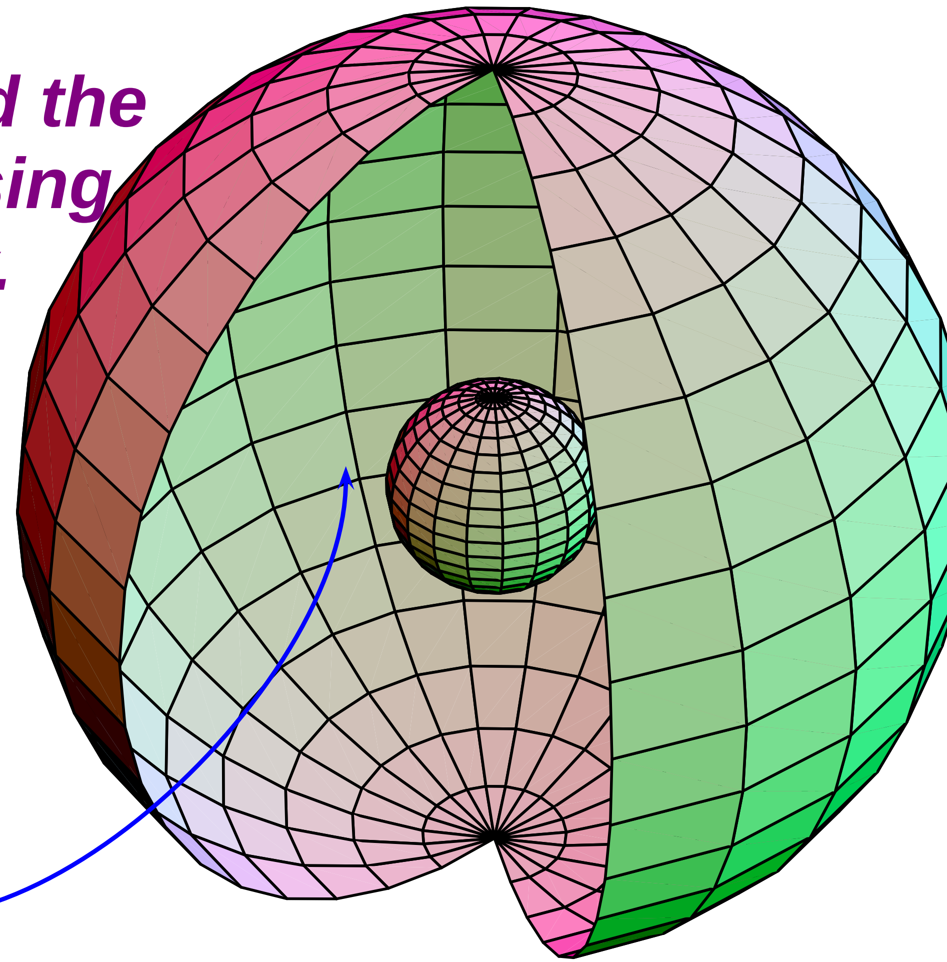


98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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- QCD – Complex behaviour

  arises from apparently simple rules

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First | Contents | Back | Conclusion

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First Contents Back Conclusion

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  - Probably not, if range range $\sim \dfrac{1}{2\, M_Q}$

First Contents Back Conclusion

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First Contents Back Conclusion

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Is a unique long-range interaction between light-quarks responsible for all this or are there an uncountable infinity of qualitatively equivalent interactions?

Gauge Theories with Massless Fermions have

CHIRAL SYMMETRY

- Helicity $\lambda \propto J \cdot p$
  - Projection of Spin onto Direction of Motion
  - For massless particles, helicity is a Lorentz invariant *Spin Observable*.
  - $\lambda = \pm$ ($\|$ or anti-$\|$ to p_μ)

Chiral Symmetry

- Chirality Operator: γ_5
  - Chiral Transformation $q(x) \to \mathrm{e}^{i\gamma_5\theta}\, q(x)$

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  - Chiral Rotation $\theta = \dfrac{\pi}{2}$
    - $q_{\lambda=+} \to q_{\lambda=+}$, $q_{\lambda=-} \to -q_{\lambda=-}$
    - Hence, a theory invariant under chiral transformations can only contain interactions that are insensitive to a particle's helicity.

Chiral Symmetry

- Chirality Operator: γ_5
  - Chiral Transformation $q(x) \to \mathrm{e}^{i\gamma_5\theta}\, q(x)$
  - Chiral Rotation $\theta = \dfrac{\pi}{4}$
    - Composite Particles: $J^{P=+} \leftrightarrow J^{P=-}$
    - Equivalent to "Parity Conjugation" Operation

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First Contents Back Conclusion

- A Prediction of Chiral Symmetry
  - Degeneracy between Parity Partners

$$N(\tfrac{1}{2}^{+}, 938) = N(\tfrac{1}{2}^{-}, 1535),$$
$$\pi(0^{-}, 140) = \sigma(0^{+}, 600),$$
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How can pion mass be so small

If quarks are so heavy?!

Propagators

- Extraordinary Effects in QCD Tied to Properties of *Dressed*-Quark and -Gluon Propagators

Quark | Gluon

$$S_f(x-y) \equiv \langle q_f(x)\bar{q}_f(y)\rangle \quad D_{\mu\nu}(x-y) \equiv \langle A_\mu(x)A_\nu(y)\rangle$$

- Describe in-Medium *Propagation Characteristics* of Elementary Particles

Propagators

- Example: Solid-State Physics
  - γ propagating in a Dense e$^-$ Gas
  - Acquires a Debye Mass

    $$m_{\rm D}^2 \propto k_F^2 :\ \frac{1}{Q^2} \to \frac{1}{Q^2 + m_{\rm D}^2}$$
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First Contents Back Conclusion

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  - γ develops an Effective-mass
  - Leads to Screening of the Interaction: $r \propto \dfrac{1}{m_D}$
- Quark and Gluon Propagators:

  Modified in a similar way -

  Momentum Dependent Effective Masses
- The Effect of this is Observable in QCD

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$$\begin{aligned} \mathbf{S}_0(p) &\to \mathrm{e}^{i\gamma_5\theta} S_0(p) \mathrm{e}^{i\gamma_5\theta} \\ &= \frac{-i\gamma \cdot p}{p^2 + m^2} + \mathrm{e}^{2i\gamma_5\theta} \frac{m}{p^2 + m^2} \end{aligned}$$

  - Symmetry Violation $\propto m$
  - $\mathbf{m} = 0$: $S_0(p) \to S_0(p)$

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  - Perturbative QCD: Vanishes if $m = 0$

Dynamical **Symmetry Breaking**

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First Contents Back Conclusion

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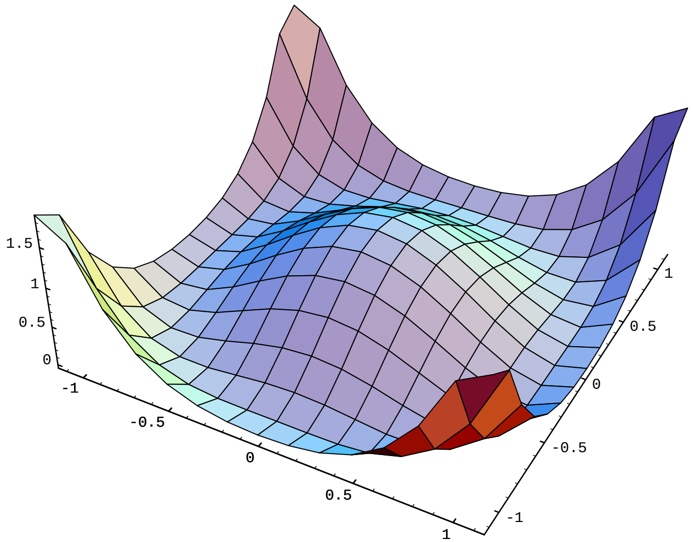

First Contents Back Conclusion

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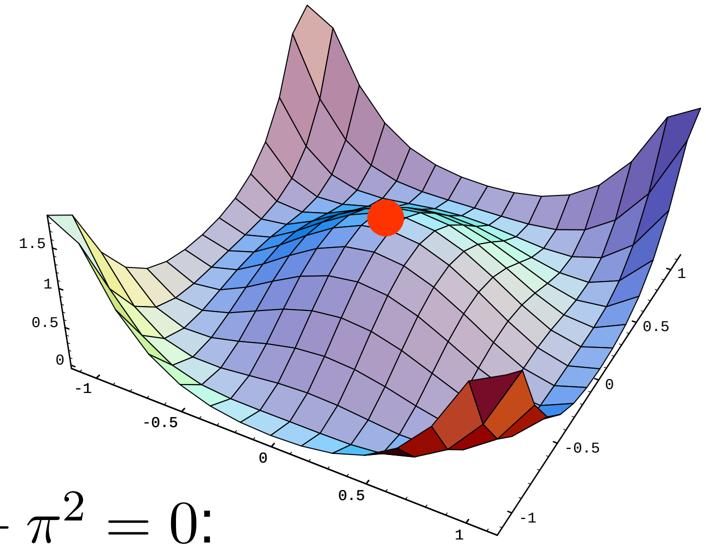

Ground State?

- Ball at (σ, π) for which $\sigma^2 + \pi^2 = 0$:
  - Rotationally Invariant

UNSTABLE

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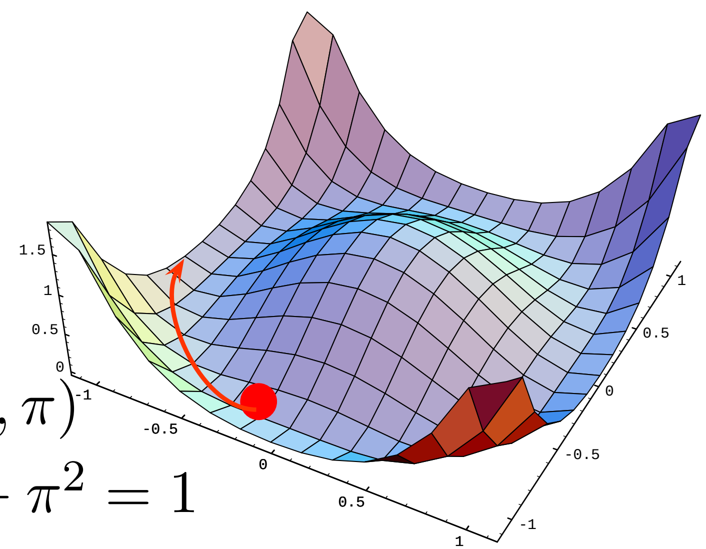

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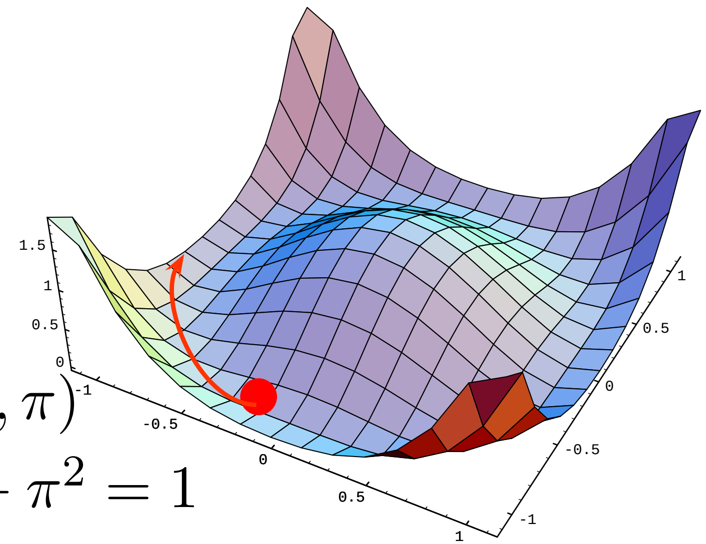
Symmetry of Ground State $\neq$ Symmetry of Hamiltonian



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First Contents Back Conclusion

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First | Contents | Back | Conclusion

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First Contents Back Conclusion

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First | Contents | Back | Conclusion

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NB. Hadron Physics Milestone, 2012: Measure the electromagnetic excitations of low-lying hadrons and their transition form factors.

Model QCD

First Contents Back Conclusion

Traditional approach to strong force problem

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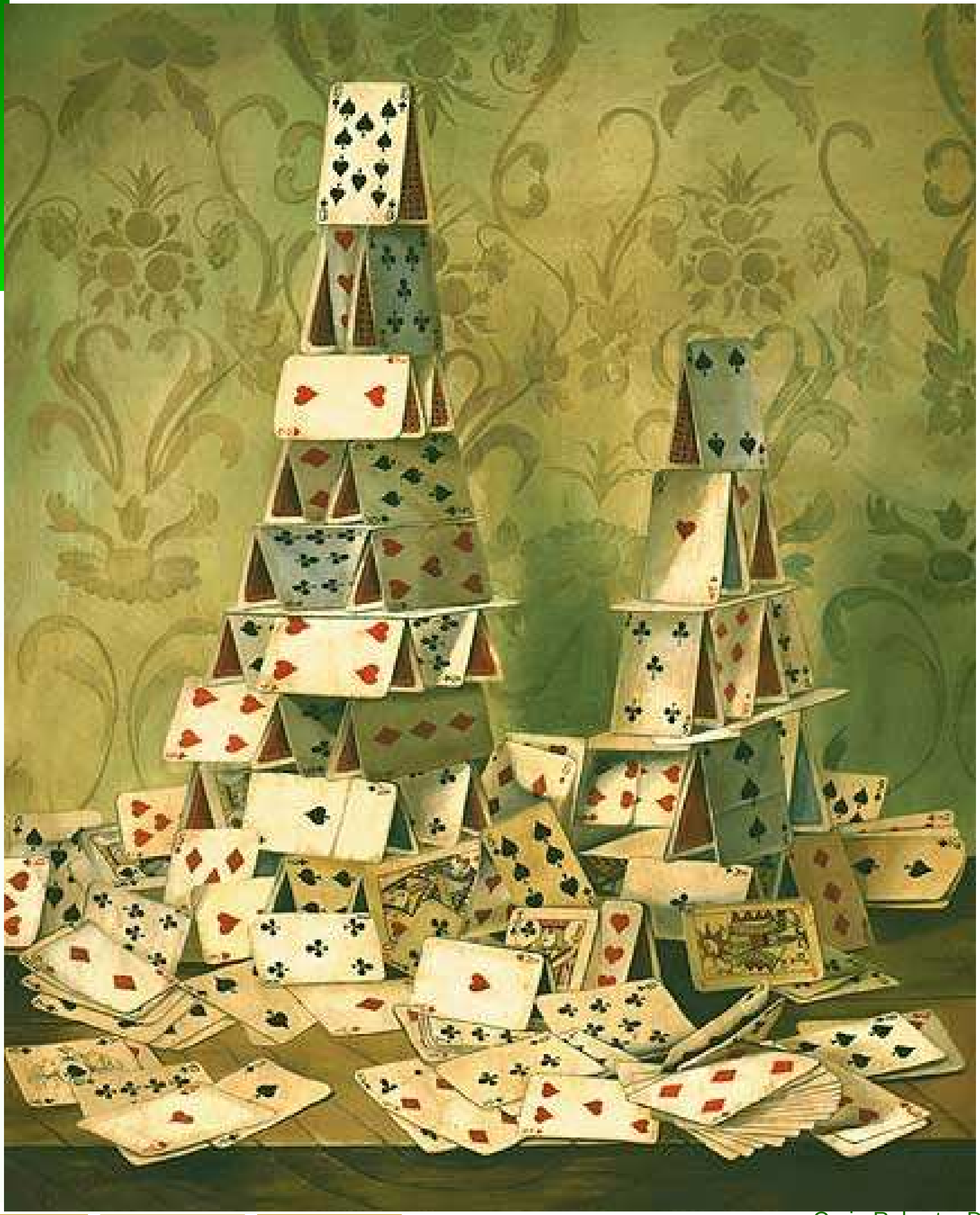
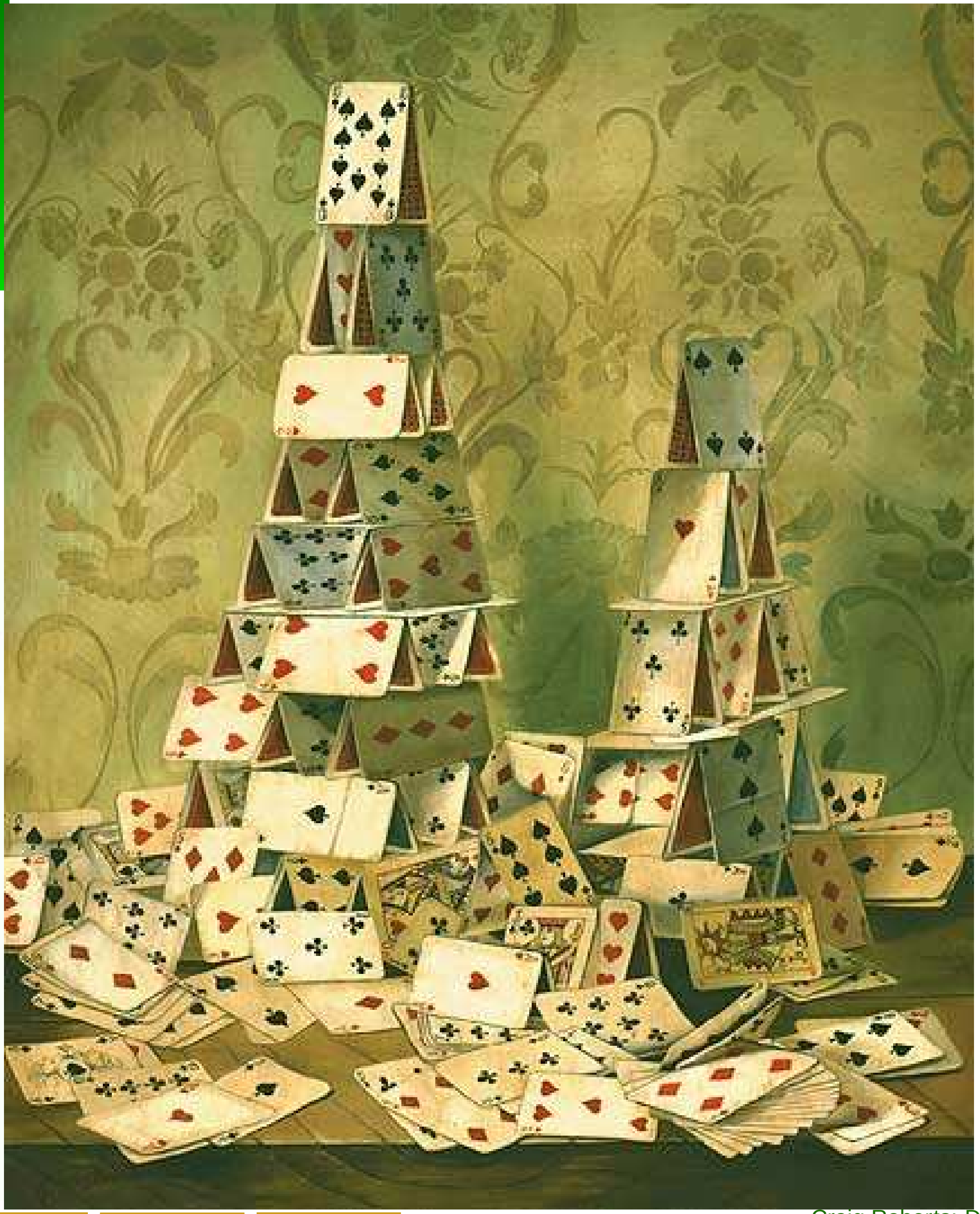
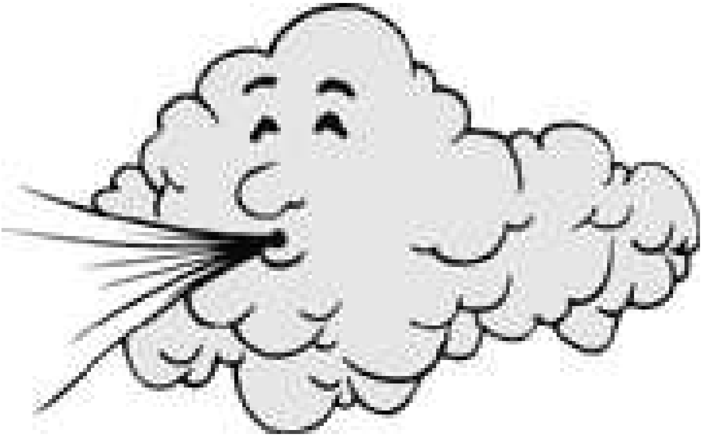


Traditional approach to strong force problem

Model QCD





Lattice QCD

One modern nonperturbative approach *Lattice QCD*

First | Contents | Back | Conclusion

One modern nonperturbative approach *Lattice QCD*

Dyson-Schwinger Equations

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- Well suited to Relativistic Quantum Field Theory

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  .... Materially Reduces Model Dependence

First Contents Back Conclusion

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First Contents Back Conclusion

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First | Contents | Back | Conclusion

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- $\Rightarrow$ Understanding InfraRed (long-range)
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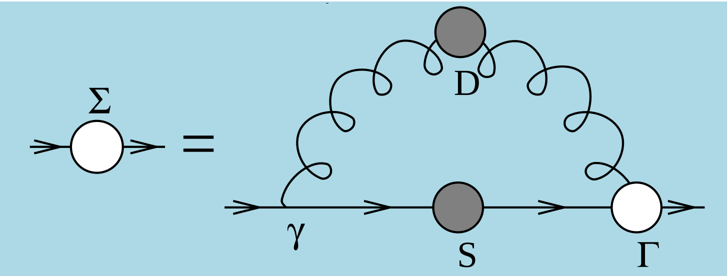
Cross-Sections built from Schwinger Functions

Perturbative Dressed-quark Propagator

First Contents Back Conclusion

Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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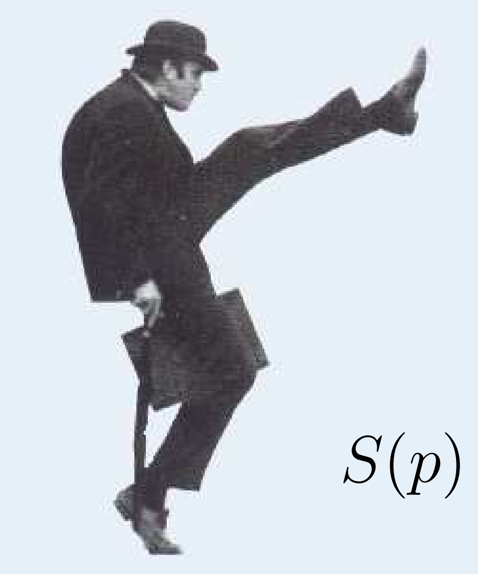

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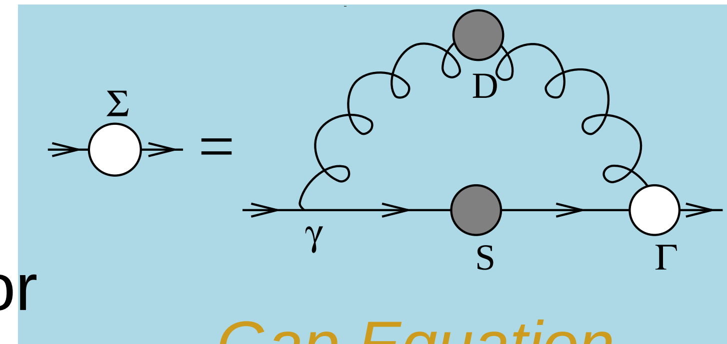

- dressed-quark propagator

Gap Equation

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

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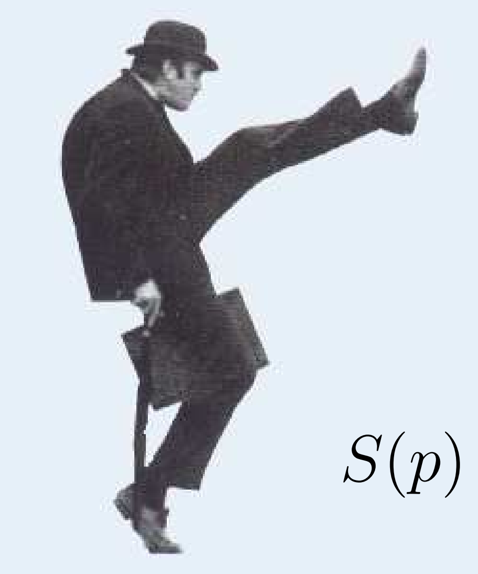

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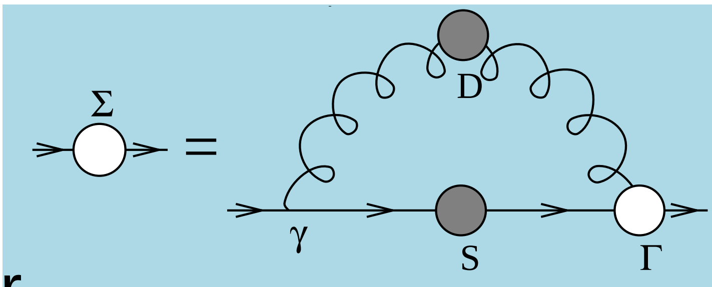

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- Weak Coupling Expansion

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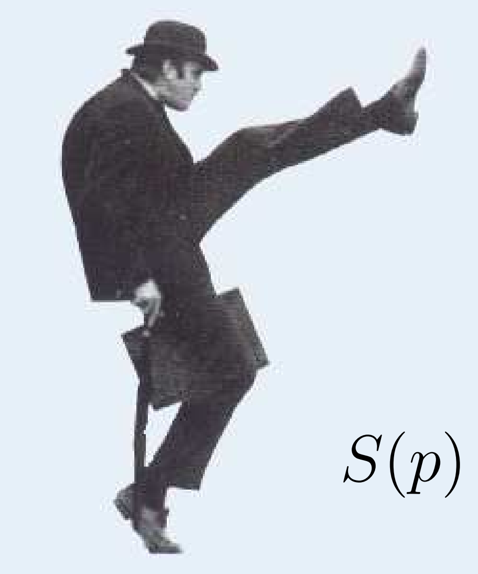

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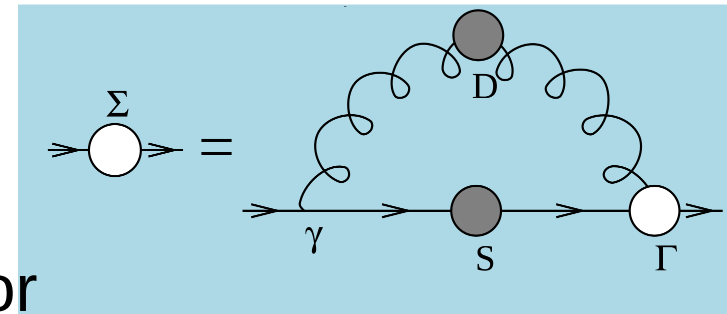

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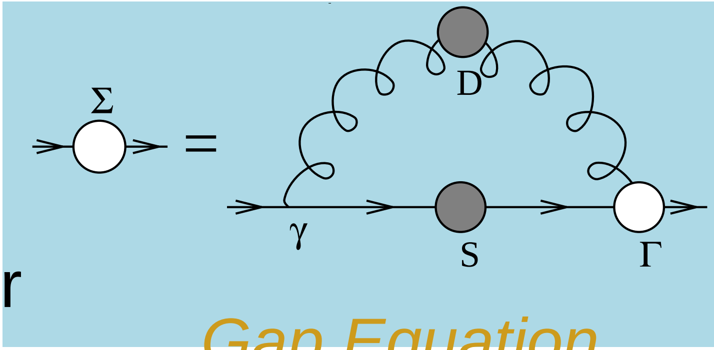
- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\to} 0$$

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

No DCSB Here!

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- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\longrightarrow} 0$$

Nambu–Jona-Lasinio Model

- Recall the Gap Equation:

$$S^{-1}(p) = i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m$$
$$+ \int^{\Lambda} \frac{d^4\ell}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-\ell)\,\gamma_\mu \frac{\lambda^a}{2} \frac{1}{i\gamma\cdot\ell A(\ell^2) + B(\ell^2)} \Gamma^a_\nu(\ell,p) \tag{4}$$

- NJL: $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \frac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(p-\ell) \to \delta_{\mu\nu}\frac{1}{m_G^2}\,\theta(\Lambda^2-\ell^2) \tag{5}$$

  - Model is not renormalisable
    $\Rightarrow$ regularisation parameter (Λ) plays a dynamical role.

- NJL Gap Equation

$$\begin{aligned} & i\gamma\cdot p\, A(p^2) + B(p^2) \\ = \quad & i\gamma\cdot p + m + \frac{4}{3}\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,\gamma_\mu \frac{-i\gamma\cdot\ell A(\ell^2) + B(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)}\,\gamma_\mu \end{aligned} \tag{6}$$

Solving NJL Gap Equation

- Multiply Eq. (6) by $(-i\gamma \cdot p)$; trace over Dirac indices:

$$p^2 A(p^2) = p^2 + \frac{8}{3} \frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4} \theta(\Lambda^2 - \ell^2)\, p\cdot\ell\, \frac{A(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)} \tag{7}$$

  - Angular integral vanishes, therefore

$$A(p^2) \equiv 1\,. \tag{8}$$

This owes to the the fact that NJL model is defined by four-fermion contact interaction in configuration space, entails momentum-independence of interaction in momentum space.

- Tracing over Dirac indices; use Eq. (8):

$$B(p^2) = m + \frac{16}{3} \frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4} \theta(\Lambda^2 - \ell^2) \frac{B(\ell^2)}{\ell^2 + B^2(\ell^2)}\,, \tag{9}$$

  - Integral is p^2-independent.
  - Therefore $B(p^2) = \text{constant} = M$ is the only solution.

NJL Mass Gap

- Evaluate integrals; Eq. (9) becomes

$$M = m + M\,\frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,\Lambda^2)\,, \quad (10)$$

$$\mathcal{C}(M^2,\Lambda^2) = \Lambda^2 - M^2\ln\left[1+\Lambda^2/M^2\right]\,. \quad (11)$$

- Λ defines model's mass-scale. Henceforth set $\boxed{\Lambda = 1}$. Then all other dimensioned quantities are given in units of this scale, in which case the gap equation can be written

$$M = m + M\,\frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,1)\,. \quad (12)$$

- Chiral limit: $m = 0$, $\boxed{M = M\,\frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,1)}$
  - Solved if $M \equiv 0$
    … This is the $\boxed{\text{perturbative result}}$: start with no mass, end up with no mass.
  - Suppose $M \neq 0$
    - Solved *iff* $\boxed{1 = \frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,1)}$.

First | Contents | Back | Conclusion

NJL Dynamical Mass

- Can one satisfy $\boxed{1 = \frac{1}{3\pi^2} \, \frac{1}{m_G^2} \, \mathcal{C}(M^2,1)}$?
- $\mathcal{C}(M^2,1) = 1 - M^2 \ln\left[1 + 1/M^2\right]$
  - Monotonically decreasing function of M
  - Maximum value at $M = 0$: $\mathcal{C}(0,1) = 1$.
- Consequently $\exists \; M \neq 0$ solution *iff* $\boxed{\frac{1}{3\pi^2} \, \frac{1}{m_G^2} > 1}$
  - Typical scale for hadron physics $\Lambda \sim 1$ GeV.
    - $M \neq 0$ solution *iff* $\boxed{m_G^2 < \frac{\Lambda^2}{3\pi^2} \simeq (0.2\,GeV)^2}$
- Interaction Strength is proportional to $\frac{1}{m_G^2}$
  - When interaction is strong enough,
    one can start with no mass but end up with a massive quark.

First Contents Back Conclusion

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Dynamical Chiral Symmetry Breaking

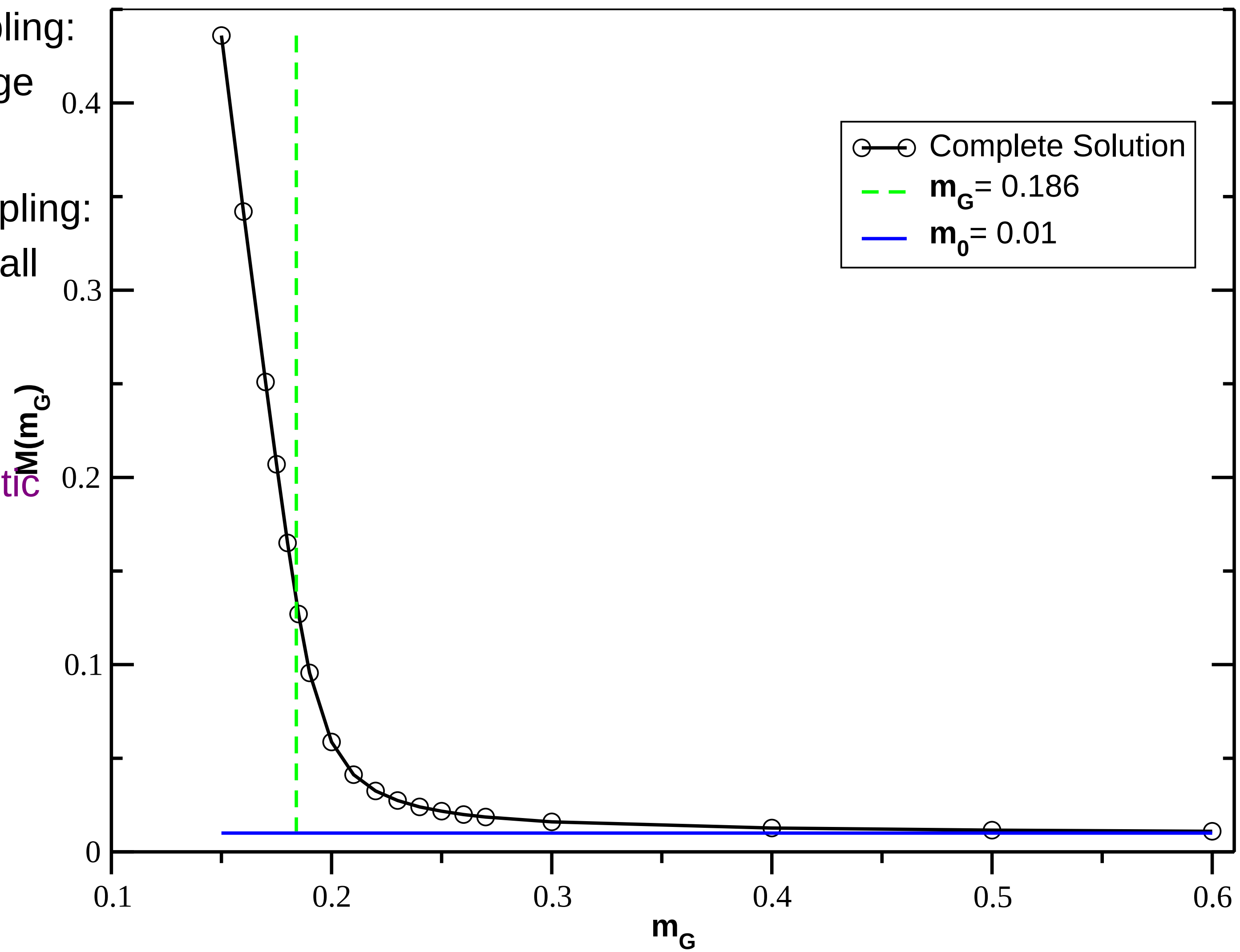
NJL Dynamical Mass

Solve $$\boxed{M = m_0 + M\,\frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,1)}$$

- Weak coupling:
  $\Leftrightarrow m_G$ large
  $M \sim m_0$
- Strong coupling:
  $\Leftrightarrow m_G$ small
  $M \gg m_0$

This is the essential characteristic of DCSB

NJL Mass Gap



NJL Model and Confinement?

- **Confinement** – no free-particle-like quarks

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- **Confinement** – no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\rm NJL} = \frac{1}{i\gamma\cdot p[A(p^2)=1] + [B(p^2)=M]} = \frac{-i\gamma\cdot p + M}{p^2+M^2} \quad (15)$$

First Contents Back Conclusion

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- This is merely a free-particle-like propagator with a shifted mass:

$$p^2 + M^2 = 0 \Rightarrow \text{ Minkowski-space mass} = M. \quad (18)$$

NJL Model and Confinement?

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- This is merely a free-particle-like propagator with a shifted mass:

$$p^2+M^2=0 \Rightarrow \text{ Minkowski-space mass} = M. \tag{20}$$

- Hence, while **NJL Model** certainly contains DCSB, it **does not exhibit confinement**.

First Contents Back Conclusion

Munczek-Nemirovsky Model

- Munczek, H.J. and Nemirovsky, A.M. (1983), "The Ground State $q\bar{q}$ Mass Spectrum In QCD," *Phys. Rev*. **D 28**, 181.
- $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \, \dfrac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(k) \to (2\pi)^4 \, G \, \delta^4(k) \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right] \tag{21}$$

Here G defines the model's mass-scale.

  - δ-function in momentum space
    cf. NJL, which has δ-function in configuration space.

- Gap equation

$$i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + G\,\gamma_\mu \, \frac{-i\gamma\cdot p\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)} \, \gamma_\mu \tag{22}$$

MN Model's Gap Equation

- The gap equation yields the following two coupled equations (set the mass-scale $G = 1$):

$$A(p^2) = 1 + 2\,\frac{A(p^2)}{p^2 A^2(p^2) + B^2(p^2)} \tag{23}$$

$$B(p^2) = m + 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,, \tag{24}$$

- Consider the chiral limit equation for $B(p^2)$:

$$B(p^2) = 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,. \tag{25}$$

  - Obviously, $B \equiv 0$ is a solution.
  - Is there another?

DCSB in MN Model

- The existence of a $B \not\equiv 0$ solution; i.e., a solution that dynamically breaks chiral symmetry, requires (in units of G)

$$p^2 A^2(p^2) + B^2(p^2) = 4\,. \tag{26}$$

- Substituting this identity into equation Eq. (23), one finds

$$A(p^2) - 1 = \frac{1}{2}\, A(p^2) \;\Rightarrow\; A(p^2) \equiv 2\,, \tag{27}$$

which in turn entails

$$B(p^2) = 2\,\sqrt{1-p^2}\,. \tag{28}$$

- Physical requirement: quark self energy is real on the spacelike domain $\Rightarrow$ complete chiral-limit solution –

$$A(p^2) = \begin{cases} 2\,; & p^2 \leq 1 \\ \frac{1}{2}\left(1+\sqrt{1+8/p^2}\right)\,; & p^2 > 1 \end{cases} \tag{29}$$

$$B(p^2) = \begin{cases} \sqrt{1-p^2}\,; & p^2 \leq 1 \\ 0\,; & p^2 > 1\,. \end{cases} \tag{30}$$

- NB. Dressed-quark self-energy is momentum dependent, as is the case in QCD.

First Contents Back Conclusion

Confinement in MN Model

- Solution is continuous and defined for all p^2, even $p^2 < 0$; namely, timelike momenta.
- Examine the propagator's denominator:

$$p^2\, A^2(p^2) + B^2(p^2) > 0\,,\ \forall\, p^2\,. \qquad (31)$$

This is positive definite . . . there are *no zeros*

- This is nothing like a free-particle propagator. It can be interpreted as describing a **confined** degree-of-freedom
- Note that, in addition there is no critical coupling: the nontrivial solution exists so long as **G** > 0.
- Conjecture: **All confining theories exhibit DCSB**.
  - NJL model demonstrates that converse is not true.

Massive Solution in MN Model

- In the chirally asymmetric case the gap equation yields

$$A(p^2) = \frac{2\,B(p^2)}{m+B(p^2)}\,, \tag{32}$$

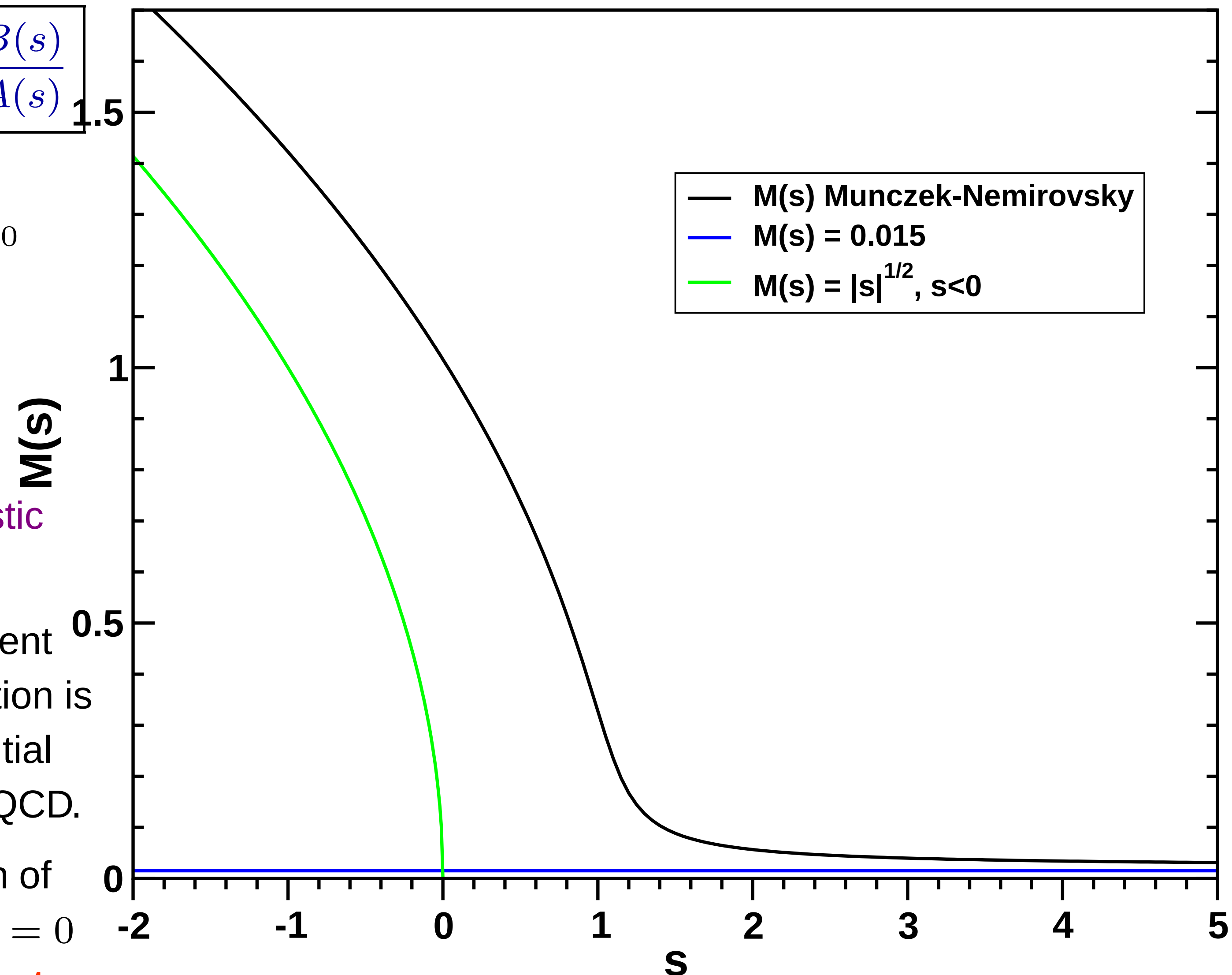
$$B(p^2) = m + \frac{4\,[m+B(p^2)]^2}{B(p^2)([m+B(p^2)]^2+4p^2)}\,. \tag{33}$$

  - Second is a quartic equation for $B(p^2)$.
  - Can be solved algebraically with four solutions, available in a closed form.
  - Only one has the correct $p^2 \to \infty$ limit: $B(p^2) \to m$.
  - NB. The equations and their solutions always have a smooth $m \to 0$ limit, a result owing to the persistence of the DCSB solution.

MN Dynamical Mass

$$M(s = p^2) = \frac{B(s)}{A(s)}$$

- Large s:
  $M(s) \sim m_0$
- Small s
  $M \gg m_0$
  This is the essential characteristic of DCSB
- p^2-dependent mass function is quintessential feature of QCD.
- No solution of
  $s + M(s)^2 = 0$
  confinement.

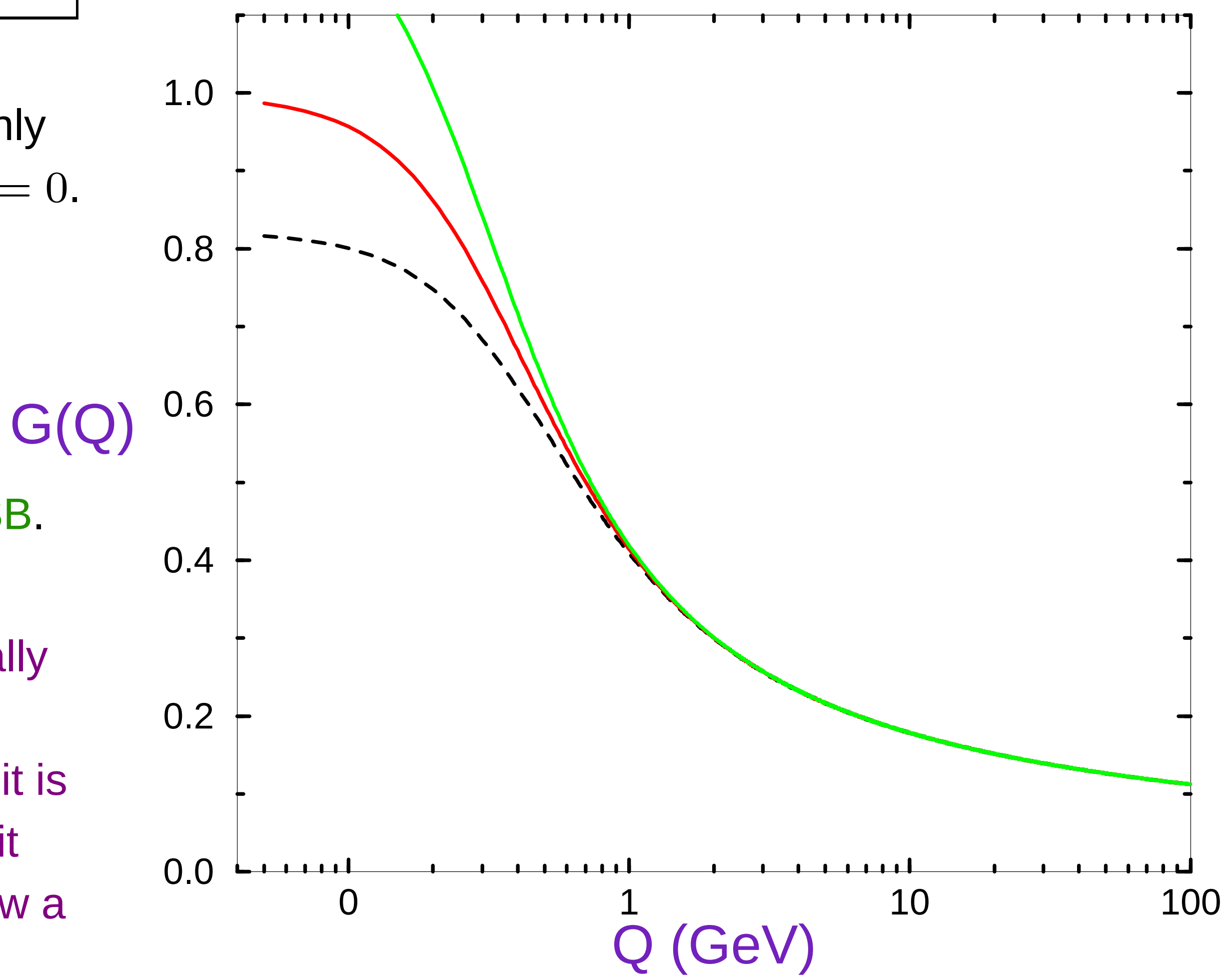


First Contents Back Conclusion

Real World Alternatives

$$\boxed{g^2 D(Q^2) = 4\pi \frac{G(Q^2)}{Q^2}}$$

- $G(0) < 1$: $M(s) \equiv 0$ is only solution for $m = 0$.
- $G(0) \geq 1$ $M(s) \neq 0$ is possible and energetically favoured: DCSB.
- $M(0) \neq 0$ is a new, dynamically generated mass-scale. If it is large enough, it can explain how a theory that is apparently massless (in the Lagrangian) possesses the spectrum of a massive theory.



Overview

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- Confinement and Dynamical Chiral Symmetry Breaking are Key Emergent Phenomena in QCD
- Understanding requires Nonperturbative Solution of Fully-Fledged Relativistic Quantum Field Theory
  - Mathematics and Physics still far from being able to accomplish that

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  - DCSB $\not\Rightarrow$ Confinement

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First Contents Back Conclusion

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First Contents Back Conclusion

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